ESSAYS IN A GENERAL EQUILIBRIUM MODEL WITH NON-COMPETITIVE MARKETS AND HETEROGENEOUS INVESTORS

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ESSAYS IN A GENERAL EQUILIBRIUM MODEL WITH NON-COMPETITIVE MARKETS AND HETEROGENEOUS INVESTORS

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This thesis investigates general equilibrium asset prices in non-competitive markets in which monopolistic traders, arbitrageurs, and extrapolators (MAX) coexist. Extrapolators form beliefs about the probability distribution of future asset prices based on sentiment, which is determined by historical asset prices. Arbitrageurs trade on mispricing but experience the limits of arbitrage, and monopolistic traders hold correct beliefs and market power. Chapter 1 provides a research overview. Chapter 2 presents a discrete-time model and investigates monopolistic traders' optimal strategies. We argue that the equilibrium price is determined not by monopolistic traders' current assets alone, but by the sequence of trades that acquired them. Monopolistic traders' decisions of placing a large block order or sequential small orders depend on both market conditions and other agents' strategies. The pump-and-dump and optimal liquidation strategies offer two examples. Results from this study explain many market phenomena, such as asset price bubbles and flash crashes, which have significant implications for financial institutions. Chapter 3 presents a continuous-time model. The model generates asset pricing characteristics, such as high equity premiums and excess volatility, while maintaining the persistence of risk-free rate and the predictability of dividend price ratio. The model proposes hypotheses and provides theoretical foundations for empirical asset pricing research, and can be used to guide profitable investment strategies.

BIOGRAPHICAL SKETCH

I am a Ph.D. in financial economics at Cornell University. My research is in the field of asset pricing and behavioral finance. I hold a master's degree in statistics and another master's degree in economics from the University of California Santa Barbara. I got the bachelor's degree in finance from Renmin University of China, with a concentration in financial engineering.

In the meantime I am a Vice President in the quantitative modeling team at Citigroup in New York. I am in charge of the liquidity risk model, which monitors the funding liquidity for the firm's over 600 billion dollar portfolio that are located in over 70 countries. I also participate in the development of various credit risk models. I apply advanced statistic models and machine learning technique to develop the model, and I use Python, R, and SAS to implement the model.

This dissertation is dedicated to my family.

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CHAPTER 1

RESEARCH OVERVIEW

This thesis examines a topic in theoretical asset pricing and behavioral finance, investigating general equilibrium asset prices in non-competitive markets with the coexistence of monopolistic traders, arbitrageurs, and extrapolators. The model relaxes traditional asset pricing model assumptions, such as rational agents and competitive markets, and was developed in both discrete- and continuous-time contexts. The discrete-time model explores optimal trading strategies, including optimal execution and market manipulation. The continuous-time model studies the relationship between asset returns and various economic conditions, generating findings regarding asset pricing, such as equity premium and excess volatility. The continuous-time model also proposes hypotheses and provides theoretical foundations to empirical asset pricing research.

The model applies a context of extrapolators similar to De Long et al. (1990b); Hong and Stein (1999); Barberis et al. (2015a,b). The investors form beliefs about the probability distribution of future asset prices based on sentiment, which is determined by historical asset prices instead of asset characteristics, and the influence decays. For example, if asset prices increased during the past several months, extrapolators expect them to keep increasing during the subsequent month. If a significant price increase occurred in the most recent month, extrapolators expect prices to also increase significantly during the next month. Extrapolators optimize consumption and investment strategies accordingly.

The innovation of this study is that it includes endogenous arbitrageurs who suffer from limits of arbitrage, which several studies discuss (Shleifer and Vishny (1997)). I model arbitrageurs' investment strategies as a mean-reverting, drift-diffusion process based on mispricing, during which mispricing is defined as the difference between fundamental and equilibrium asset prices. Fundamental asset prices are obtained by solving a general equilibrium model with rational investors alone. The equilibrium asset prices are calculated endogenously using the underlying general equilibrium problem. The convergence rate serves as a liquidity indicator, and the diffusion term serves as noise information.

The primary contribution of this study is the introduction of monopolistic traders and an endogenous price impact function, whereas extant literature commonly assumes an exogenous function. Jarrow (1992) examines large traders' roles during market manipulation by specifying a convex price impact function exogenously. Different from extant research and acknowledging that price impact functions essentially describe the supply and demand relationship of an underlying asset, I apply the market clearing condition in the general equilibrium model as a price impact function. I therefore relax the competitive market assumption by having monopolistic traders incorporate market clearing conditions in their optimization problem. Monopolistic traders also incorporate other agents' strategies when making investment decisions.

The discrete-time model derives general equilibrium asset prices as a function of monopolistic traders' entire trading history. Monopolistic traders can thus exploit irrational investors and form optimal trading strategies based on their objective functions. I discuss optimal liquidation and market manipulation strategies as two examples. Monopolistic traders' choice of a large block order or sequential small orders depends on market conditions, and profitability varies across markets. I also show that in a plain economy without a fundamental shock and zero initial sentiment, no market manipulation strategy is profitable.

The continuous-time model simultaneously reproduces many characteristics of asset pricing. Market power enables monopolistic traders to require higher expected returns, which aligns with the equity premium puzzle. Asset price volatility is reinforced by extrapolators' sentiment and arbitrageurs' noise information, which explain the excess volatility puzzle. The model also proposes testable assumptions, providing a theoretical foundation for several empirical asset pricing studies. The first assumption suggests that low extrapolator sentiment predicts high asset returns. Results from Baker and Wurgler (2006, 2007); Huang et al. (2015) corroborate this assumption. The second assumption suggests that changes to institutional ownership predict asset returns positively, with several empirical studies supporting this assumption (Asquith et al. (2005); Sias et al. (2006)). The continuous-time model also helps investigate additional topics by varying economic conditions and analyzing investors' performance.

This study assesses the general equilibrium model with heterogeneous investors in non-competitive markets. The model covers most types of players in financial markets—irrational and rational investors, informed and uninformed investors, passive and active investors, and individual and institutional investors. The model generates optimal trading strategies, corroborating several asset pricing characteristics. The model also proposes hypotheses and provides a theoretical foundation for many empirical asset pricing studies. In future research, the model should be extended to a multi-asset model, and market data should be assessed to explore its profitability. For example, researchers should analyze the predictive power of extrapolators' sentiment under varying institutional ownership markets and liquidity conditions. Fama–French type empirical techniques should also be applied in this case.

CHAPTER 2

CAN MONOPOLISTIC TRADERS EXPLOIT IRRATIONAL TRADERS?

2.1 Introduction

Standard asset pricing theory focuses on competitive markets in which traders are price-takers who optimize utility. The equilibrium price is thus established by the market clearing condition, and such models assume perfect elasticity so orders of arbitrary size do not affect asset prices. In non-competitive markets, traders influence asset prices and therefore must consider the influence of their optimization strategies. Using a discrete-time, general equilibrium model in a non-competitive market, this paper investigates monopolistic traders' optimal strategies with the coexistence of two realistic and typical types of agents—arbitrageurs and extrapolators. Arbitrageurs trade on price deviations from fundamental value, believing that deviations will be corrected in the future. In traditional asset-pricing literature, they are treated as rational traders. Extrapolators form beliefs about expected price changes based on weighted averages of past price changes, a type of agent documented well in behavioral finance literature. Price impacts cause additional costs because orders are executed only after prices have been adjusted adversely. Although they must always trade on the adverse side, monopolistic traders can use their market power to derive optimal strategies by making rational assumptions and having full knowledge about other agents' behaviors.

Some studies (Hong and Stein (1999); Barberis et al. (1998)) characterize extrapolators' behaviors as overreactions to a sequence of good news. De Long et al. (1990a) propose an overlapping generation model of noise traders whose irrational assumptions persist into a subsequent period, creating additional risks for asset prices. Greenwood and Shleifer

(2014) assess the extrapolation of stock market returns, finding that many investors believe that stock prices will continue to rise after they have risen and continue to fall after they have fallen. Choi and Mertens (2006) argue that extrapolators' overreactions to dividend news generate counter-cyclical expected returns that explain the equity premium puzzle. Hirshleifer et al. (2015) show that extrapolative bias explains many stylized facts about financial markets, such as high equity premiums, volatile stock returns, and low and smooth risk-free rates. Barberis et al. (2015a) develop an equilibrium model with interactions of fundamental traders and extrapolators, deriving extrapolators' optimal strategies as a linear function of sentiment. The current paper applies this context to model extrapolators whose optimal strategies are a linear function of sentiment, adjusted for risk appetite, in which sentiment is defined as weighted averages of past price changes. The model explains how heterogeneous agents interact and monopolistic traders exploit extrapolators.

This study contributes to literature that examines large traders and price influences in non-competitive markets. Extant studies commonly assume an exogenous price impact function. Jarrow (1992) emphasizes the role of large traders during market manipulations, providing examples of the price impact function. Frey and Stremme (1997) extend this idea to a continuous-time model. Liu and Yong (2005) model the evolution of asset prices according to a jump-diffusion process, in which price impact is characterized by the jump component. Jonsson et al. (2004) assume that the price impact function is exponential, and other studies, including Chevalier et al. (2013), Vath et al. (2007), Cvitanić et al. (1996), Henderson and Hobson (2011), Rogers and Singh (2006), and Løkka (2014), assume that price impacts take various exogenous forms. Different from these models, the current equilibrium model has a price impact function that is endogenous, and consequently, it accords with Jarrow (2016), who derived the first competitive market equilibrium asset pricing model with endogenous liquidity risk. The current model deviates from Jarrow (2016) by deriving the price impact function from the market clearing condition, which specifies the supply–demand relationship in the underlying economy. By incorporating arbitrageurs' and extrapolators' optimal strategies, we obtain a relationship between equilibrium price and monopolistic traders' positions. This function resembles either a supply or demand function because monopolistic traders' positions influence both sides. We then incorporate the knowledge of price impact function into monopolistic traders' optimization problem to derive the equilibrium price. It is an equilibrium model because each agent is optimal and the market clears, and the price impact function is embedded in the general equilibrium model.

Literature on large traders usually specifies price impact as either temporary or permanent. Bank and Baum (2004) assume temporary price changes, demonstrating the absence of arbitrage for large traders. Cetin et al. (2004) and Jarrow and Protter (2005) also assume temporary price changes while investigating arbitrage. Khemchandani et al. (2013) and Bertsimas and Lo (1998) include a permanent component in their model. We develop this idea by specifying price impact as a combination of temporary and permanent components. The temporary component is arbitrageurs' reactions to price deviations and extrapolators' sensitivities to current price change, and the permanent component is extrapolators' sentiment. The weight is shifted to investigate strategies across economies.

Consistent with market microstructure literature, price impact represents liquidity cost, in which magnitude indicates a market's depth. Due to a need for immediacy, investors' orders are often larger than the quantity of shares available at the best market quote. Hence, to achieve immediate execution, it is often necessary to dig into the limit order book, which increases the price of each successive transaction. When the price impact is significant, searching for liquidity around the current price is difficult, and traders must search for a more unfavorable quote in the limit order book. When the price impact is negligible, there is abundant liquidity around the prevailing quote. Chevalier et al. (2013) present a model accordingly, in which price impacts on the bid and ask prices are modeled separately, and optimal market strategies are obtained. Jarrow (2016) and Amihud (2002) assume liquidity risk is a pricing factor in multiple-factor models, and we extend this idea by assuming that a portion of liquidity risk is manipulative, created by the strategies of monopolistic traders. Therefore, liquidity risk is considerable only to arbitrageurs and extrapolators, depending on their strategies.

This paper also contributes to asset pricing literature by deriving a general equilibrium model that includes three heterogeneous agents. Traditional asset pricing models commonly focus on one representative agent or two heterogeneous agents. Wang (1993) introduces a rational expectation equilibrium model in which the asset price is endogenous, deriving a linear relationship between the asset price and state variables. Many studies follow this idea and obtain similar linear relationships in both continuous- and discrete-time contexts. Models with more than two agents are usually too complicated to obtain closed-form solutions. Detemple et al. (2014) derive a dynamic, noisy, rational expectation model that includes informed and uninformed investors, and active unskilled investors, but assume that unskilled investors have a functional form similar to that of informed traders; they mimic informed traders' strategies. In the current model, traders' strategies are disparate. De Long et al. (1990b) use a three-period parsimonious model to show market manipulation strategies and bubble creation with the coexistence of feedback and passive investors. They argue that these strategies and their destabilizing effects have prevailed, for example, in the Dutch tulip bulb market bubble and George Soros' riding of conglomerates. However, in their model, feedback investors and passive investors are specified exogenously. In our model, both extrapolators and arbitrageurs derive optimal strategies to maximize their utility functions, and closed-form solutions of their strategies and equilibrium prices are derived.

This study also contributes to market manipulation literature. Allen and Gale (1992) divide manipulation into three categories—information-, action-, and trade-based. During information-based manipulation, false information is released or rumors are spread, and action-based manipulation resembles an activism strategy. These two types of manipulation are monitored strictly by the U.S. Securities and Exchange Commission. However, the third type of manipulation, trade-based, is difficult to eradicate. This type of manipulation is defined as manipulating the market simply by buying and selling without taking publicly observable actions to alter the value of a firm, or releasing false information to change the price. Both Jarrow (1992) and De Long et al. (1990b) provide examples of trade-based manipulation in a three-period model. The current study demonstrates that with the existence of extrapolators, the endogenous price impact function, and favorable market conditions, trade-based manipulation is profitable in a multi-period economy, and monopolistic traders are able to manipulate price despite always trading on the adverse side. The logic is intuitive. When monopolistic traders place a buy order, which drives up asset prices, they expect an asset price to further increase because extrapolators' sentiments also increase. When monopolistic traders place a sell order, the price might not decrease significantly if extrapolators' sentiments remain high. This study also relates to asset price bubbles. Barberis et al. (2015a) show that bubbles can be generated by a sequence of good news. The current study shows that bubbles can also be generated by purely trade-based manipulation with fundamental shocks and investor sentiment. A reasonable conjecture is that the coexistence of a sequence of good news and trade-based manipulation has stimulated several significant bubbles throughout history. Other strategies, such as optimal execution, can be demonstrated similarly.

2.2 Model

We propose a discrete-time, heterogeneous-agent model in which some traders extrapolate past price changes when forecasting future price changes, while other traders are rational in the sense that they trade on price deviations from fundamental value. The equilibrium asset price is determined by each trader's optimal strategy and the market clearing condition. We deviate from traditional literature by assuming monopolistic traders have superior knowledge about other traders' strategies. They are also aware that their trades influence asset prices, and therefore take advantage by incorporating the market clearing condition or price impact function into their optimal strategies.

2.2.1 Assets

We consider a discrete-time economy with infinite horizon $t \in \{0, 1, ..., \infty\}$. We assume complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\{0,1,...,\infty\}}, \mathbb{P})$ in which $\mathbb{P}$ is the statistical probability measure. The economy consists of two assets—one riskless and one risky. The riskless asset earns a constant return, which we normalize to zero. The risky asset, which has a fixed supply of Q shares, has fundamental value

$$P_t^r = P_0^r + \epsilon_1 + \epsilon_2 + ... + \epsilon_t$$

, where

$$\epsilon_i \sim N(0, \sigma_\epsilon^2), i.i.d.$$

P_0^r is the risky asset's fundamental value at time 0. Innovation ϵ_i is normally distributed with a mean of zero and constant variance σ_ϵ^2, where the variance is assumed greater than zero. The i_{th} innovation ϵ_i becomes public information between time $i-1$ and i. The risky asset price at time t, P_t is determined implicitly by equilibrium.

2.2.2 Agents

We assume three types of agents in this economy—rational arbitrageurs, irrational extrapolators, and monopolistic traders. The first two have identical utility functions but different beliefs about the probability distributions of future asset prices; they represent fundamental and technical investors. Monopolistic traders are smart and strategic in the sense that they are aware of other agents' preferences and beliefs.

Arbitrageurs

The first type of agent, arbitrageurs, believes that a current mispricing will be corrected during the subsequent period, and consequently trade on the difference between current market prices and expected fundamental values. During each period, arbitrageurs maximize a CARA utility function defined over the subsequent period's wealth:

$$\max_{N_t^a} \mathbb{E}_t^a \left[-e^{-\gamma(W_t^a + N_t^a(\tilde{P}_{t+1} - P_t))} \right]$$

W_t^a is their wealth at time t, and N_t^a is arbitrageurs' demand at time t. The utility maximization problem gives the time t optimal strategy:

$$N_t^a = \frac{P_t^r - P_t}{\gamma \sigma_\epsilon^2}$$

where $P_t^r = P_0 + \sum_{i=1}^t \epsilon_i$ is the fundamental price of a risky asset at time t. A proof appears in Appendix A.1. Arbitrageurs are rational traders under traditional asset pricing theory, but their rationality is bounded in the sense that they are myopic; they are unaware of other agents' strategies. Without mispricing, they are absent from the market, so the no-trade theorem applies. With mispricing, they consider other players as noise traders who trade for exogenous reasons. Arbitrageurs, who believe they don't have market power, act as price-takers.

Extrapolators

The second type of agent, extrapolators, is also myopic. They maximize a CARA utility function defined over the subsequent period's wealth:

$$\max_{N_t^e} \mathbb{E}_t^e \left[-e^{-\gamma(W_t^e + N_t^e(\tilde{P}_{t+1} - P_t))} \right]$$

W_t^e represents their wealth at time t, and N_t^e is extrapolators' demand at time t. However, they are different from arbitrageurs because they form expectations about the subsequent period's price change according to past price changes. Their expected price change is:

$$\begin{aligned} X_t^e &= \mathbb{E}_t^e(\tilde{P}_{t+1} - P_t) \\ &= (1-\theta) \sum_{k=1}^{t-1} \theta^{k-1} (P_{t-k} - P_{t-k-1}) + \theta^{t-1} X_1^e + \theta^t X_0^e \end{aligned}$$

where X_t^e is the expected price change in extrapolators' beliefs at time t. This relationship also serves as a measure of investors' sentiments in behavioral finance literature. We henceforth denote X_t^e as sentiment. X_0^e and X_1^e measure extrapolators' initial sentiments.

Solving the utility maximization problem yields extrapolators' optimal strategy:

$$N_t^e = \frac{X_t^e}{\gamma \sigma_\epsilon^2}$$

This specification indicates that extrapolators' demand is linear in sentiment, which accords with the continuous-time MAX-Capital Asset Pricing Model (MAX-CAPM). We apply a similar context of extrapolators to Barberis et al. (2015a). The specification of sentiment is similar to previous models of extrapolative beliefs (Cutler et al. (1990); Hong and Stein (1999); Barberis and Shleifer (2003); Barberis et al. (2015b)). Some studies also call extrapolators feedback traders (De Long et al. (1990b)). As with arbitrageurs, extrapolators behave like price-takers.

Monopolistic Traders

Monopolistic traders maximize expected utility based on objectives. Examples include large-order execution and market manipulation. In non-competitive market, with price impact, monopolistic traders do not enjoy a current price before orders are placed. Their orders are instead exercised after a price is influenced by orders, suffering from liquidity cost. Cetin et al. (2004) and Bank and Baum (2004) consider the difference between paper and liquidation values. We apply the same spirit to monopolistic traders' problem. The optimization problem at time t for monopolistic traders with trading horizon $T - t + 1$ is described as:

$$\max_{\{N_t^m,\dots,N_T^m\}} \mathbb{E}_t^m \left[-e^{-\gamma[W_t^m + f(N_t^m, N_{t+1}^m, \dots, N_T^m)]} | P_0(N_0^m), P_1(N_0^m, N_1^m), \dots, P_T(N_0^m, N_1^m, \dots, N_T^m) \right]$$

subject to

$$g_i[N_0^m, N_1^m, \dots, N_T^m | P_0(N_0^m), P_1(N_0^m, N_1^m), \dots, P(N_0^m, N_1^m, \dots, N_T^m)] = 0, i = 1, 2, \dots, K$$

where W_t^m is monopolistic traders' deterministic initial wealth at the beginning of time t. N_t^m is their demand at time t. $f(N_t^m, N_{t+1}^m, \dots, N_T^m)$ is their objective, and g_is are constraints that reflect their self-financing trading strategies with initial wealth.

Monopolistic traders are funds and institutional investors who commonly experience various time constraints. Funding liquidity risk is paramount, but problems force them to pay much attention to period reports. Consequently, we assume monopolistic traders have a finite horizon. Although they operate in an infinite horizon economy, they enter the market at time t with a certain initial wealth, and they accomplish their objectives and exit the market at time T. Typical monopolistic traders' problems are discussed below.

Example 1: Optimal Liquidation Strategies

Suppose a broker holds a positive initial position of risky asset, and he or she wants to close this position within a trading window of T periods. The objective is to maximize liquidation value, or minimize the difference between paper and liquidation value. The problem is:

$$\max_{\{N_k^m\}_{(t \le k \le T)}} \mathbb{E}_t^m \left[-e^{-\gamma[N_{t-1}^m (P_{t-1} + \epsilon_t) + (\sum_{k=t}^T P_k (N_{k-1}^m - N_k^m))]} \right]$$

subject to

$$\begin{aligned} 0 &= N_{t-1}^m - m \\ 0 &= N_T^m \end{aligned}$$

Example 2: Optimal Buying Strategies

Suppose a broker starts with initial funds W_t^m. He or she uses a sequence of buy orders to enter the market, aiming to optimize paper wealth at time T. The buy order drives up asset prices, and the order is executed after price change. Therefore, the broker experiences a paradox. If he or she executes a large block order, the asset becomes very expensive, but if a sequence of small orders is executed, both more fundamental shocks and time constraints are experienced. The problem is:

$$\max_{\{N_k^m\}_{(t\le k\le T)}} \mathbb{E}_t^m \left[-e^{-\gamma(W_t^m+P_T N_T^m)}\right]$$

subject to

$$\begin{aligned} 0 &= \sum_{k=t}^{T} P_t(N_t^m - N_{t-1}^m) - W_t^m \\ 0 &= N_{t-1}^m \end{aligned}$$

Example 3: Market Manipulation Strategies

Suppose a market manipulator has market power, and he or she applies trade-based strategies to maximize cash wealth. These strategies include pump-and-dump and bear raid. The problem is:

$$\max_{\{N_k^m\}_{(t\le k\le T)}} \mathbb{E}_t^m \left[-e^{-\gamma[(\sum_{k=t}^{T} P_k(N_{k-1}^m - N_k^m))]}\right]$$

subject to

$$\begin{aligned} 0 &= N_{t-1}^m \\ 0 &= N_T^m \end{aligned}$$

2.2.3 Market Clearing

Assuming the portion of arbitrageurs, extrapolators, and monopolistic traders are μ^a, μ^e, and $\mu^m = 1 - \mu^a - \mu^e$, respectively, the market clearing condition is:

$$\mu^a N_t^a + \mu^e N_t^e + \mu^m N_t^m = Q$$

In non-competitive financial markets with large traders and market power, a price impact function is assumed, a demand and/or supply function that indicates the relationship between large traders' positions and market prices. In our equilibrium model, the market clearing condition serves as the endogenous price impact function, which is known by only monopolistic traders who take advantage of the knowledge. Plugging in arbitrageurs' and extrapolators' optimal strategies yields a clearer supply–demand relationship:

$$P_t = P_t^r - \frac{\gamma\sigma_\epsilon^2 Q}{\mu^a} + \frac{\mu^e}{\mu^a} X_t^e + \frac{\mu^m \gamma\sigma_\epsilon^2}{\mu^a} N_t^m$$

in which extrapolators' sentiments have a recursive dynamic:

$$X_t^e = \begin{cases} (1-\theta)(P_{t-1} - P_{t-2}) + \theta X_{t-1} & t > 1 \\ X_t^e & t = 0, 1 \end{cases}$$

When writing recursively, extrapolators' sentiments consist of two components - the previous sentiment and the most recent price changes. θ measures the relative weight. When θ is low, sentiment is determined primarily by the most recent price changes, and when high, even price changes in the distant past affect a current sentiment. Absent fundamental value shocks, the sentiment decays exponentially by proportion θ. A lemma gives a closed-form representation of extrapolators' sentiments in terms of monopolistic traders' strategies and exogenous fundamental shocks.

Lemma 1 *Extrapolators' sentiments at time t are determined endogenously by their initial sentiments, the realized fundamental value shocks before time t, and monopolistic traders' sequences of trades before time t, expressed as:*

$$
\begin{aligned}
X_k^e = {} & \frac{a^k - b^k}{a - b} X_1^e - ab \frac{a^{k-1} - b^{k-1}}{a - b} X_0^e \\
& + (1 - \theta)\eta \sum_{i=1}^{k-1} \frac{a^i - b^i}{a - b} (N_{k-i}^m - N_{k-i-1}^m) \\
& + (1 - \theta) \sum_{i=1}^{k-1} \frac{a^i - b^i}{a - b} \epsilon_{k-i}
\end{aligned}
\tag{2.1}
$$

where $\eta = \frac{\mu^m \gamma \sigma_\epsilon^2}{\mu^a}$. a and b are roots of a second-order polynomial that is specified in Appendix A.2.

The lemma suggests that monopolistic traders can use their sequences of trades to manipulate extrapolators' sentiments. Two factors determine the significance of influence on sentiments—the magnitude of a trade and the time when the trade occurs. Applying the lemma gives a closed-form representation of asset price.

Proposition 1 *The asset price, or the price impact function, is a function of monopolistic traders' sequences of trades:*

$$
\begin{aligned}
P_t = {} & P_t^r - \frac{\gamma \sigma_\epsilon^2 Q}{\mu^a} + \frac{\mu^e}{\mu^a} \left[\frac{a^t - b^t}{a - b} X_1^e - ab \frac{a^{t-1} - b^{t-1}}{a - b} X_0^e \right] \\
& + (1 - \theta)\eta \frac{\mu^e}{\mu^a} \sum_{i=1}^{t-1} \frac{a^i - b^i}{a - b} (N_{t-i}^m - N_{t-i-1}^m) + \eta N_t^m \\
& + (1 - \theta) \frac{\mu^e}{\mu^a} \sum_{i=1}^{t-1} \frac{a^i - b^i}{a - b} \epsilon_{t-i}
\end{aligned}
\tag{2.2}
$$

where $\eta = \frac{\mu^m \gamma \sigma_\epsilon^2}{\mu^a}$. a and b are roots of a second-order polynomial that is specified in Appendix A.2.

The market clearing condition gives a supply–demand relationship. The asset price is determined by total supply Q and agents' total demands, which are themselves determined by the asset price. Arbitrageurs trade on the difference between the current price and fundamental value, and extrapolators trade on their sentiments. Besides the exogenous shocks, both incentives can be influenced by monopolistic traders' sequences of trades, and consequently, the asset price is also determined by the joint effect of exogenous shocks and monopolistic traders' trading histories. The further away trades or shocks occurred, the less influence they have on the current price. The price impact function is unknown by either arbitrageurs or extrapolators since they do not have market power and act as price-takers. However, monopolistic traders are knowledgeable, aware of this relationship, and consider it during their optimization problem. We next define monopolistic traders–arbitrageurs–extrapolators (MAX) equilibrium.

2.2.4 Equilibrium

In the economy described above, MAX equilibrium is defined as price process $\{P_t\}_{t\geq 0}$ and trading strategies (N_t^a, N_t^e, N_t^m) such that:

1. The trading strategies solve each agent's optimization problem at each time t.
2. The market clears such that supply equals demand or asset price process $\{P_t\}_{t\geq 0}$ is as described in Proposition 1.

Arbitrageurs and extrapolators are myopic since their utility functions are defined only over the next period's wealth. Contrarily, monopolistic traders' utility functions are defined over their objectives, which are over the entire trading horizon. To achieve equilibrium at each time t, they must incorporate realized information and solve the optimization problem continuously. For example, at time t, monopolistic traders form trading plan $\{N_t^m, ..., N_T^m\}_t$ for trading horizon $\{t, t+1, ..., T\}$. They hold $(N_t^m)_t$ of risky asset at time t according to the trading plan. At time $t+1$, public information ϵ_{t+1} becomes available and is observed by all traders. At this point, monopolistic traders do not necessarily follow the time t plan and hold $(N_{t+1}^m)_t$ of the risky asset. They instead construct updated trading plan $\{N_{t+1}^m, ..., N_T^m\}_{t+1}$ for trading horizon $\{t+1, t+2, ..., T\}$. They then hold $(N_{t+1}^m)_{t+1}$ of the risky asset accordingly. They incorporate updated information when creating trading strategies at time $t+1$ such that the trading horizon decreases by one, the asset price at the beginning becomes $P_t + \epsilon_{t+1}$, and extrapolators' sentiments become X_t^e.

Most finance literature that examines non-competitive markets treats the price impact function as exogenous, though the degree is determined endogenously; the more shares purchased, the more impact on the asset price. In our model, the price impact function is embedded in the equilibrium condition. Since monopolistic traders' strategies ensure the market clearing condition, their optimization problem leads directly to MAX equilibrium.

2.3 Optimal Execution

The optimal execution problem has many market applications, particularly when an investor must buy or liquidate asset positions within a fixed horizon. Suppose a hedge fund must cover a short position at time T. The task is thus to buy, for example, one percent of total market shares before the due date. Due to liquidity constraints, it might be costly to place the entire order at the same time. The hedge fund can instead consider forming an optimal buying strategy and spread the orders over the trading horizon. In a similar example, a broker wants to liquidate some large block orders to minimize a client's costs.

Many studies assess price impact, and most assume a large trader and exogenous price impacts (Frey and Stremme (1997); Cetin et al. (2004); Bank and Baum (2004); Rogers and Singh (2006)). MAX equilibrium applies a different approach. No exogenous price impact function is assumed, and exogenous fundamental value shocks can be zero or go either way.

2.3.1 MAX Equilibrium for Optimal Liquidation Strategy

In this section, the optimal liquidation strategy is used as an example. Monopolistic traders are treated as brokers who hold a positive initial position in a risky asset at time 0, aiming to close the position within a trading window of T periods. The trader wants to minimize the difference between the initial paper and final liquidation value. Since selling large volumes aggressively saves time but substantially reduces the asset price, the trader must find a trade-off between a large block order and a sequence of small orders. The broker has superior knowledge about other traders' strategies, of which they can take advantage.

Monopolistic traders' optimal liquidation strategy at time 1 is:

$$\max_{\{N_k^m\}_{(1\leq k\leq T)}} \mathbb{E}_1^m \left[-e^{-\gamma[N_0^m(P_0^r+\epsilon_1)+(\sum_{k=1}^T P_k(N_{k-1}^m - N_k^m))]} \right]$$

subject to

$$\begin{aligned} 0 &= N_0^m - m \\ 0 &= N_T^m \end{aligned}$$

Proposition 2 *Given information available at time* 1*, under MAX equilibrium, monopolistic* *traders' optimal trading strategy* $\boldsymbol{N_1^m} = \{N_1^m, ..., N_T^m\}_1$ *at time* 1 *is:*

$$\boldsymbol{N_1^m} = \boldsymbol{A_1^{-1}}\boldsymbol{b_1} \tag{2.3}$$

where $\boldsymbol{A_1}$ *is* $(T-1)\times(T-1)$ *matrix:*

$$\boldsymbol{A_1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1(T-1)} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2(T-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{(T-1)1} & a_{(T-1)2} & a_{(T-1)3} & \cdots & a_{(T-1)(T-1)} \end{bmatrix}$$

and $\boldsymbol{b_1}$ *is* $(T-1)\times 1$ *vector:*

$$\boldsymbol{b_1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{T-1} \end{bmatrix}$$

Expressions of $\boldsymbol{A_1}$ *and* $\boldsymbol{b_1}$ *appear in Appendix A.2.*

Proposition 2 provides monopolistic traders' optimal liquidation strategy at time 1 based on the information available. This problem concerns convex optimization. The following corollary states the sufficient condition for the existence of MAX equilibrium:

Corollary 1 *Monopolistic traders' optimal liquidation strategy and MAX equilibrium exist if the diagonal elements in* $\boldsymbol{A_1}$ *are non-positive.*

Expressions of the diagonal elements in $\boldsymbol{A_1}$ appear in Appendix A.2, and a proof is in Appendix A.2.

2.3.2 Numerical Example

To illustrate the results in proposition 2, we consider three numerical examples in which trading horizon T equals 3, 4, and 5, respectively. We follow values of the continuous-time MAX-CAPM paper and assume that risk aversion γ is 0.1, fundamental shock volatility σ is 0.25, and total supply of the risky asset Q is 5. We assume that there are no realized fundamental shocks, so the time 1 trading plan equals the realized trading plan. We also assume that extrapolators' initial sentiments are zero. Monopolistic traders' initial position N_0^m is 3 and their market share is 40%. They therefore must liquidate 24% of the total share of the risky asset. Market shares of the arbitrageurs and extrapolators are set to 30%. θ equals 0.75, so the current price change contributes to 25% of the current sentiment.

Figure A.1 shows the trading plan and equilibrium price in each case. Across a longer trading horizon, monopolistic traders sell more slowly, and therefore the liquidation cost, defined as the difference between the initial paper and terminal liquidation value, decreases as the trading horizon increases. Selling reduces the asset price, which recovers gradually after monopolistic traders exit the market. Figure A.2 shows the trading plan and equilibrium price when θ is 0.25, so the current price change contributes to 75% of the current sentiment. Extrapolators are particularly sensitive to the current price change, and in this case, monopolistic traders sell more aggressively. For example, in the three-period model, they sell 80% of their shares at time 1 in comparison to 67% in the previous case. The liquidation cost increases in all three cases, which demonstrates that this market is less favorable to monopolistic traders. The asset price plunges more substantially in comparison to the previous case, but then recovers rapidly.

In most cases, monopolistic traders sell less aggressively across a longer trading horizon. However, one unexpected observation, shown in panel (c) of figure A.2, was that monopolistic traders placed one block order and sold 93% of their shares at time 1. As the asset price plunged, the arbitrageurs traded to correct the mispricing. The asset price then recovered rapidly from time 2. Extrapolators' sentiment also recovered. With superior knowledge about other traders' strategies, the monopolistic traders took advantage of the price recovery. They placed a large buy order at time 3, followed by a sell order at time 4 to clear their position. The last two trades generated extra profit and reduced the liquidation cost. In comparison to the high θ case, the liquidation cost increased by only 0.0033 during the five-period model, in contrast to 0.0095 during the three-period model and 0.0117 during the four-period model.

Figure A.3 shows the trading plan and equilibrium price in a scenario of an initial fundamental shock. We reset θ to 0.75. In response to a negative fundamental shock, the asset price plunged rapidly. Monopolistic traders reacted by placing a large block order at time 1. However, they still incurred a tremendous liquidation cost. In another scenario and in response to a positive fundamental shock, monopolistic traders sold more slowly. They sold 37% of their shares at time 1 in response to a positive fundamental shock, in comparison to 87% in response to a negative fundamental shock. The liquidation cost reduced by 50%. Therefore, the prime timing for liquidation was immediately after positive news.

Figure A.4 panel (a) shows the trading plan and equilibrium price when the initial sentiment was negative. The negative initial sentiment drove the asset price below the fundamental value. Arbitrageurs were expected to correct the mispricing, which led to an upward price trend. The positive price change increased extrapolators' demand, which further increased the asset price. In this scenario, monopolistic traders waited one period until a price trend was created. From time 2, they were able to hide their sell orders;

they liquidatedwithout moving the asset price in an unfavorable direction. Finally, their liquidation value exceeded the initial paper value, which was due to the increased asset price. Hence, monopolistic traders were able to both liquidate their position and gain positive profit.

Figure A.4 panel (c) shows the trading plan and equilibrium price when the initial sentiment was positive. The asset price was expected to decrease over the trading horizon. Monopolistic traders' best strategy was to sell the entire position immediately. They also short sold to mitigate their losses. However, in the optimal trading strategy, they still incurred a huge liquidation cost.

Our model considers the optimal execution strategy under various market conditions. A favorable market condition includes a positive initial fundamental shock and negative initial sentiment. Extrapolators' attributes also played a role in the strategy.

2.4 Market Manipulation

In this section, monopolistic traders are treated like market manipulators, with trade-based manipulations possible. A typical manipulation strategy is pump-and-dump, during which an upward price trend is created and then traded against. Other strategies include the bear raid and short squeeze. Jarrow (1992) provides a sufficient condition in the asset price process, which excludes market manipulation trading strategies—the asset price process depends only on manipulators' aggregated holdings, not the sequence of trades that attained it. This sufficient condition is violated by the coexistence of manipulators, arbitrageurs, and extrapolators, and consequently, market manipulation is possible in the economy. However, manipulators must face the paradox of manipulation.

They can use their market power to manipulate asset prices in their favor, but their orders are exercised only after prices have adjusted, so they must always trade on the adverse side. The second point means that when they place an order and the price changes, they enjoy only the changed price. Whether they are able to profit is questionable, depending on the economy.

2.4.1 MAX Equilibrium for Market Manipulation Strategy

During trade-based market manipulation, information is available to all traders. Monopolistic traders hold zero shares of a risky asset at time 0. They apply the optimal trading strategy to maximize their real wealth, defined as the total amount of cash collected over the trading horizon. They exit the market at time T, when their position of the risky asset returns to zero. Monopolistic traders' optimal market manipulation strategy at time 1 is:

$$\max_{\{N_k^m\}_{(1\leq k\leq T)}} \mathbb{E}_1^m \left[-e^{-\gamma[(\sum_{k=1}^{T} P_k(N_{k-1}^m - N_k^m))]}\right]$$

subject to

$$\begin{aligned} 0 &= N_0^m \\ 0 &= N_T^m \end{aligned}$$

Proposition 3 *Given information available at time* 1*, under MAX equilibrium, market manipulators' optimal trading strategy* $\boldsymbol{N_1^m} = \{N_1^m, ..., N_T^m\}_1$ *at time* 1 *is:*

$$\boldsymbol{N_1^m} = \boldsymbol{X_1^{-1}} \boldsymbol{y_1} \tag{2.4}$$

where $\boldsymbol{X_1}$ *is* $(T-1)\times(T-1)$ *matrix:*

$$\boldsymbol{X_1} = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1(T-1)} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2(T-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{(T-1)1} & x_{(T-1)2} & x_{(T-1)3} & \cdots & x_{(T-1)(T-1)} \end{bmatrix}$$

and $\boldsymbol{y_1}$ *is* $(T-1)\times 1$ *vector:*

$$\boldsymbol{y_1} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{T-1} \end{bmatrix}$$

Expressions of $\boldsymbol{X_1}$ *and* $\boldsymbol{y_1}$ *appear in Appendix A.2, and a proof appears in Appendix A.2.*

A corollary provides the sufficient condition to ensure MAX equilibrium in the economy.

Corollary 2 *Monopolistic traders' optimal market manipulation strategy and MAX equilibrium exist if the diagonal elements in* $\boldsymbol{X_1}$ *are non-positive.*

A question is raised of whether pure trade-based manipulation is profitable in a plain economy without a fundamental shock and with zero initial sentiment. A corollary addresses this question.

Corollary 3 *In a plain economy without a fundamental shock and with zero initial sentiment, no market manipulation strategy is profitable. A manipulator's optimal strategy is not to trade.*

In a plain economy, monopolistic traders' optimal strategy is to hold zero shares of the asset. Extrapolators hold zero shares of the asset because the sentiment is zero. The market is efficient because the equilibrium price reflects the fundamental value, which excludes arbitrageurs' activity from the market. Hence, no trade theorem applies.

2.4.2 Numerical Example

However, market manipulation is profitable when an exogenous disturbance drives the market value away from the fundamental value. In this case, the optimal strategy and the ability to profit varies according to market conditions. To illustrate the results of proposition 3, we provide examples in which trading horizon T equals 3, 4, and 5. In line with the previous section, we assume risk aversion γ is 0.1, fundamental shock volatility σ is 0.25, and total supply of the risky asset Q is 5. Market shares of monopolistic traders, arbitrageurs, and extrapolators are set at 40%, 30%, and 30%, respectively. We assume no realized fundamental shocks after time 1, so the time 1 trading plan is the realized trading plan.

Figure A.5 shows the trading plans and equilibrium prices across trading horizons when there is a positive initial fundamental shock. θ is set to 0.75, so 75% of the previous sentiment is inherited and the current price change contributes to 25% of the current sentiment. In each case, monopolistic traders follow a pump-and-dump strategy to create a positive price trend and trade against it. Hence, a positive asset price bubble is created. Monopolistic traders' profit increases by 20% when the trading horizon increases from 3 to 5 periods. When the trading horizon is sufficiently long, monopolistic traders take advantage of the downward price trend and sell short during the last period to make extra profit.

We examine the case in which extrapolators pay more attention to the current price change. Results are shown in figure A.6. θ is set to 0.5, so the current price change contributes to 50% of the current sentiment. A larger asset price bubble is created, which is 20% greater than the fundamental value, in comparison to 10% in the previous case. The optimal market manipulation strategy is still pump-and-dump, but manipulators are able to generate much larger profit, as much as $5-6$ times greater than in the previous case. Manipulators prefer sensitive extrapolators and high market volatility.

In this example, the larger bubble is accompanied by a higher trading volume. The trading volume is nearly three times higher than the trading volume in the previous case. That bubbles feature high trading volumes is pointed out in finance literature Ofek and Richardson (2003); Carlos et al. (2006); Hong and Stein (2007). Our model explains this feature. Figure A.7 shows the trading plans and equilibrium prices across trading horizons when there is a negative initial fundamental shock. Without short-sell constraints, the bear raid strategy is stimulated, and a negative price bubble is created. Initially, the market crashes, which is followed by a slow recovery. However, since monopolistic traders are able to sell short, their profitability is not weakened by the negative shock.

Figure A.8 shows the trading plans and equilibrium prices under various initial sentiments. We assume that sentiment at time 0 is negative. Figure A.8 panel (a) shows that the sentiment at time 1 decreases and deviates further from zero. Absent monopolistic traders, decreasing sentiment creates a downward price trend, followed by an upward trend as the negative sentiment gradually diminishes and arbitrageurs correct the mispricing. Monopolistic traders react by placing a buy order at time 1 to slightly support the price. They do not push the price to the fundamental level because they want to take advantage of the subsequent upward trend and thereby exit the market secretly.

In this case, they trade aggressively and gain a large profit.

Shown in figure A.8 panel (b), we assume that at time 1, sentiment does not change from time 0. A flat yet upward price trend is created. Monopolistic traders' strategy is similar to the case shown in panel (a), but they trade less aggressively, which generates less trading volume. Hence, profit is only one-third of that shown in panel (a). In panel (c), we assume that at time 1, sentiment becomes slightly positive. Monopolistic traders' demand and profit decrease significantly. When sentiments swing widely, they offset each other, which decreases monopolistic traders' profit. When no extrapolators are in the market, no market manipulation strategy is profitable. In this case, monopolistic traders' optimal strategy is not to trade. Therefore, they cannot exploit rational arbitrageurs, and they profit only in the presence of extrapolators, whose sentiment reacts to the price impact.

2.5 Conclusion

This paper examines the equilibrium price of a financial market using three typical types of agents—monopolistic traders, arbitrageurs, and extrapolators. The equilibrium price is determined by two factors. The first is the current market condition, which includes, for example, asset fundamental risk, aggregated supply, risk aversion, and extrapolators' attitudes toward past price changes. The second is monopolistic traders' past sequences of trades. The equilibrium asset price is determined by not only monopolistic traders' current holdings, but trading history. Results have implications for institutional investors who have a strong research ability and are therefore aware of the behaviors of other types of agents in the market. They are also aware that their trades influence the market price.

Considering their objectives, our model helps them construct optimal strategies.

For example, a broker has a large initial position and wants to liquidate it after T periods. When the broker starts to sell, he or she considers whether arbitrageurs will correct the price over time. He or she must also consider whether extrapolators will sell, which might amplify the downside price movement. Using our model, the broker is able to decide whether to liquidate the position using a large block order or hide the sell order in a sequence of small trades to maximize the liquidation value. Another example is market manipulation, which continues to be popular especially in emerging markets. In China's A-share market, over 60% of investors are retail investors who lack basic financial knowledge, so they are manipulated easily. Their behaviors resemble that of extrapolators in our model. A typical market manipulation strategy is pump-and-dump. When manipulators begin to exit the market, they must ensure that extrapolators will think the price is still rising, so they take over sell orders. Our model helps monopolistic traders create market manipulation strategies. We demonstrate that a purely trade-based market manipulation strategy is unprofitable in the absence of fundamental shocks and extrapolator sentiment.

We assume that the fundamental price of a risky asset is affected only by *i.i.d* shocks. A natural extension of this model is to add complicated structures to the asset's fundamental price. One determinant might be the asset's fundamental risk, and another the market's risk-bearing capacity. Structures should be added to the arbitrageurs' strategies. For example, in addition to market liquidity risk, arbitrageurs also face funding liquidity risk and short-sale constraints. In this case, they trade less aggressively when there is a large price bubble. Short-sale constraints cause a bubble to expand (Xiong and Yu (2011)), which strengthens monopolistic traders' pump-and-dump strategy.

In the real world, multiple strategic, rational, monopolistic traders coexist in the market. Profitability depends not only on their rationality, but the ability to study other traders' strategies. A trader who knows his or her competitors profits most. To corroborate this theory, another type of monopolistic trader should be considered, one aware of arbitrageurs and extrapolators but is unaware of the existence of other monopolistic traders. The profitability of both types of monopolistic traders should be studied in future research. The model is based on a single risky asset. In the real world, thousands of assets correlate, and optimal trading strategies are commonly based on dependency between assets. The model should be extended to a multi-asset model in which empirical data generate trading strategies to corroborate the model. Parameters should be calibrated to represent current market conditions. Econometric models could also be used to assess optimal trading window T. Although this model is designed for application to median-frequency trading, traders can also apply it to high-frequency trading contexts.

CHAPTER 3

THE MONOPOLISTIC TRADERS, ARBITRAGEURS, AND EXTRAPOLATORS CAPITAL ASSET PRICING MODEL

3.1 Introduction

A primary research focus in finance is assessing determinants of cross-sectional and time-series properties of asset prices. Since Breeden (1979), consumption asset pricing models have been introduced that use marginal rates of substitution to determine the stochastic discount factor of risky asset prices. However, empirical tests on the original model are largely negative. Cochrane (2009) argues that contemporary theoretical research into the behaviors of aggregated risky asset prices have simultaneously incorporated four empirical facts. First, Mehra and Prescott (1985) find the equity premium puzzle, and Mehra (2003) shows that the average post-war real return on the S&P500 index is 8.4% per year, while the less risky security return is 0.6% per year, which leads to very large equity premiums – of about 7.8% returns annually. This requires an unreasonably large risk-aversion parameter in the original model. Second, LeRoy and Porter (1981); Shiller (1981) identify the excess volatility puzzle. Stock returns are very volatile, with a standard deviation of 17% per year[1]. Third, post-war household data suggest that risk-free rates are persistent. Fourth, dividend price ratios are excellent predictors of the percentage of long-term stock returns (Fama and French (1988); Campbell and Shiller (1988a,b)). Although earlier research (Campbell and Shiller (1988a); Hansen and Singleton (1982, 1983)) uses time-varying discount rates to explain these puzzles, later research emphasizes time-varying risk-aversion. The most popular models are the habit formation (Campbell and

[1]Gilchrist (2013): lecture notes from Boston University

Cochrane (1999)) and long-run risk models (Bansal and Yaron (2004)).

The original model applies several assumptions. First, all investors are rational and have identical probability beliefs for states of the world. Second, markets are competitive and frictionless, and agents act as price-takers. In the real world, investors incorporate disparate market strategies, and not all strategies are rational, or they are only rational under some subjective probability measures. According to Greenwood and Shleifer (2014), evidence supports extrapolation of the expectations of stock market prices, and despite fundamental values, extrapolative investors expect future stock prices to increase or decrease after increases or decreases to past prices. Arbitrageurs are also not always able to correct mispricing, though their trades move prices closer to fundamental values. They dampen irrational price movements but do not commonly eliminate them. For example, they are subject to market liquidity risk, funding liquidity risk, and information noise. Institutional investors are common in financial markets. According to the New York Stock Exchange Factbook, institutional investors increased their market shares from 4.3% in 1950 to 42.9% in 2010. Most institutional investors build knowledgeable and experienced research teams to explore competitors' strategies, and they are aware that their trades influence market prices. They are no longer price takers but monopolistic traders who have market power in non-competitive markets.

This paper extends the consumption-based asset pricing model by relaxing the identical rational agents and competitive market assumptions. Using a monopolistic traders, arbitrageurs, and extrapolators capital asset pricing model (MAX-CAPM), we investigate general equilibrium asset prices in non-competitive markets with heterogeneous agents. The model was developed in a continuous-time framework with standard constant absolute risk aversion (CARA) utility and an identical risk aversion parameter across agents. General equilibrium risky asset prices are obtained in closed form by applying a dynamic programming technique and solving high-order partial differential equations (PDEs). Parameters of the PDEs are calculated by solving a system of polynomials, and in this way, each agent's optimal investment strategy is determined.

The existence and behaviors of extrapolators have been investigated by several researchers. Hong and Stein (1999) and Barberis et al. (1998) incorporate extrapolators in their models, and De Long et al. (1990a) propose an overlapping generation model with irrational noise traders whose incorrect beliefs persist during subsequent periods, creating additional risk. Similar to the heterogeneous-agent context in this paper, De Long et al. (1990b) introduce a model with positive feedback traders, passive investors, and informed rational speculators. Their discrete time, three-period model suggests that market power creates bubbles, generating positive returns. Greenwood and Shleifer (2014) survey expectations of stock market returns, finding that many investors believe that stock prices continue to increase after having previously increased, and decrease after having previously decreased. Barberis et al. (2015b) present a consumption capital asset pricing model that includes interactions between rational traders and extrapolators, in which sentiment is a state variable. Choi and Mertens (2006) argue that extrapolators' overreactions to dividend news generate counter-cyclical expected returns that explain the equity premium puzzle. Hirshleifer et al. (2015) also use extrapolative bias to explain stylized facts about financial markets, such as high equity premiums, volatile stock returns, and low risk-free rates.

This paper models extrapolators in a way similar to Barberis et al. (2015b). Investors form probability beliefs about future asset prices based on sentiment, determined by historical asset prices instead of asset characteristics, and the influence of historical asset prices decays. For example, if asset prices have been increasing for several months, extrapolators expect an asset price to continue increasing during the subsequent month.

If a significant price increase occurs, extrapolators expect the asset price to increase significantly in the near future, and they optimize their strategies accordingly. In our model, extrapolators' incorrect beliefs also create incorrect perceptions of arbitrageurs' behaviors, which has an amplified effect on their decisions.

The constraints arbitrageurs experience are documented well in finance literature. De Long et al. (1990a) and Shleifer and Vishny (1997) model such constraints using a context in which irrational investors are optimistic and pessimistic. Shleifer and Vishny (1997) show that across short horizons, arbitrageurs lose money when pessimism persists, especially when they experience agency problems. De Long et al. (1990b) argue that risk aversion prevents arbitrageurs from correcting mispricing over time, and Xiong and Yu (2011) find that short-sell constraints create abnormal price movements and bubbles. Other constraints include market liquidity, funding liquidity, and information. Wang (1993) discusses the information precision effects of asset prices, and market microstructure literature uses information, inventory, and liquidity to explain constraints during price formation. Our model follows these studies, but its innovation is that it includes the endogenous equilibrium asset prices while maintaining the limits of arbitrage. Arbitrageurs' investment strategies are modeled using mean-reverting drift diffusion based on mispricing, in which mispricing is the difference between fundamental and equilibrium asset prices. Fundamental asset prices are obtained by solving a general equilibrium model with only rational investors, and equilibrium asset prices are calculated endogenously using the underlying general equilibrium problem. Convergence serves as the liquidity indicator, and diffusion a measure of information precision.

This paper relates to research on large traders and price impacts in non-competitive markets. However, extant literature commonly assumes an exogenous price impact function. For example, Jarrow (1992) studies the role of large traders during market manipulation by specifying a convex price impact function. Frey and Stremme (1997) extend the idea to a continuous time model, and Cetin et al. (2004) assume a stochastic supply curve for a security price as a function of trade size. Liu and Yong (2005) model the evolution of asset prices as a jump-diffusion process, during which the price impact is characterized in the exogenous jump component. Jonsson et al. (2004) assume that the price impact function takes an exogenous exponential form. Other papers, including Chevalier et al. (2013), Vath et al. (2007), Cvitanić et al. (1996), Henderson and Hobson (2011), Rogers and Singh (2006), and Løkka (2014), assume that the price impact takes various exogenous forms.

In contrast, one contribution of the current study is the introduction of monopolistic traders and an endogenous price impact function. This paper and Jarrow (2016) are first to endogenize the price impact function in the equilibrium asset pricing model. In this paper, the price impact function describes the supply–demand relationship of an asset; when supply or demand changes, the equilibrium asset price also changes. Therefore, the market clearing condition in the general equilibrium model serves as a price impact function, and the equilibrium price is determined endogenously by equating supply and demand. The competitive market assumption is thus relaxed by having monopolistic traders incorporate the market clearing condition during their optimization problem. A rational expectation equilibrium is obtained because each agent is able to guess the correct functional form of the equilibrium price and then verify this initial guess after solving the model.

We build the MAX-CAPM in a continuous-time environment. The portfolio selection problem using continuous-time model is introduced by Merton (1971). In this model, a dynamic programming technique and the Hamilton-Jacobi-Bellman (HJB) equations are applied to obtain PDEs that characterize the behaviors of each agent's value function. Parameters of the PDEs are calculated by solving a system of fifteen polynomials, among which six are derived from extrapolators' optimization problem, six from monopolistic traders' optimization problem, and three from the market clearing condition. The solution to the system of polynomials describes the solution to the model completely.

Heterogeneous agents and a non-competitive market context simultaneously reproduce many characteristics in empirical asset pricing data. Market power enables monopolistic traders to require higher expected returns, which justifies the equity premium puzzle. Expected returns increase alongside the current bearishness of sentiment, the limits of arbitrage, and the market shares of monopolistic traders. Extrapolators' wealth diminishes, and they are continuously replaced by the next generation of extrapolators. The volatility of asset prices increases when extrapolators are more sensitive to current price changes, or when arbitrageurs' information is less precise. This result explains the excess volatility puzzle. The risk-free rate is set endogenously by the central bank at the beginning to maximize initial consumption, and is persistent thereafter. The expected return is determined by the relative dividend and equilibrium prices, which aligns with the predictive power of the dividend price ratio.

The continuous-time model suggests several hypotheses that provide theoretical foundations for empirical asset pricing research. The first assumption is that low sentiment predicts high asset returns, and results from Baker and Wurgler (2006, 2007); Huang et al. (2015) corroborate the assumption. The second assumption is that changes to institutional ownership predict asset returns positively, with several studies, including Jones et al. (1999), Nofsinger and Sias (1999), Wermers (1999), Bennett et al. (2003), and Parrino et al. (2003), supporting the assumption. By introducing a method of generating higher frequency covariances, Sias et al. (2006) investigate the source of this predictive power and provide two explanations. First, institutional investors are better informed and hold correct beliefs about the distribution of future asset prices. Second, institutional investors' trades have direct effects on asset prices through the supply–demand relationship. We assume that monopolistic traders hold correct beliefs about risky asset prices and the market clearing condition. We show that increases to monopolistic traders' market shares increase expected returns. Both assumptions and results corroborate results from Sias et al. (2006). Other assumptions concern time-series momentum, market liquidity, and information precision. The continuous-time model can also be used to investigate additional topics by varying economic conditions and analyzing investors' performance.

3.2 The Benchmark Model

We introduce a homogeneous agent rational expectation model, which serves as a benchmark and provides fundamental values of a risky and riskless asset. Risky asset prices and returns are determined by a rational expectation equilibrium, and the risk-free rate is set endogenously by the central bank at the beginning to maximize initial consumption, and is persistent thereafter. The context of rational agents is the same as that in the traditional asset pricing model. Since a random component exists, the asset price may deviate from the fundamental value. Rational agents' trading strategies serve as a correction mechanism, which guarantees that the price does not deviate after incorporating rational agents' preferences. A heterogeneous-agent model is then introduced with a natural assumption that the fundamental values of the risky and riskless assets enter agents' decision-making; each agent forms an optimal strategy that depends on results from the benchmark model, which is also helpful while analyzing comparative statics.

3.2.1 The Setup

Consider a continuous-time, infinite-horizon economy in which uncertainty is represented by complete probability space $(\Omega, \mathbb{F}, \mathbb{P})$ and information filtration $(\mathbb{F}_t)_{t\geq 0}$, for which $\mathbb{F}_t$ is the information set observed up to time t by rational agents. The economy includes two assets. A risky asset has a fixed supply of Q shares, and its equilibrium price at time t is denoted P_t^r. The risky asset generates an instantaneous dividend of D_t, which is governed by diffusion process

$$dD_t = g_D dt + \sigma_D dZ_t^D$$

, where g_D is the deterministic dividend growth rate and σ_D the volatility. We assume that they are both positive and constant. dZ_t^D is a Brownian motion that characterizes the randomness of the dividend process. The riskless asset has constant rate of return r ($r \geq 0$). This risk-free rate is set by the central bank at time 0 to maximize the consumption (i.e., utility) at that time. Knowing the dividend process, rational agents choose optimal consumption and investment strategies to maximize the expected utility at time 0 as

$$\mathbb{E}_0^r\left[-\int_0^\infty \frac{e^{-\delta t-\gamma C_t^r}}{\gamma}dt\right]$$

subject to

$$dW_t^r = (rW_t^r - C_t^r)dt + N_t^r[(D_t - rP_t^r)dt + dP_t^r]$$

for which γ is the risk aversion parameter and δ the time discount factor. Being fully informed in an economy, rational agents correctly conjecture probability beliefs for future dividends. The optimal strategy is determined by asset prices and dividends, and the risky asset price is determined endogenously by rational agents' strategies and market clearing conditions.

3.2.2 Equilibrium

Rational expectation equilibrium is defined as consumption and investment strategy (C_t^r, N_t^r) and price process P_t^r, such that:

1. Each agent chooses the consumption and investment strategy to maximize his or her expected utility.
2. The price clears the security market (i.e., $N_t^r = Q$).

Proposition 4 *In an economy with homogeneous rational agents, equilibrium risky asset prices are*

$$P_t^r = p_0^r + \frac{D_t}{r}$$

where

$$p_0^r = \frac{g_D - \gamma Q \sigma_D^2}{r^2}$$

The optimal consumption and investment strategy is

$$\begin{aligned} C_t^r &= rW_t^r - \frac{r - \delta}{r\gamma} + \frac{\gamma \sigma_D^2 Q^2}{2r} \\ N_t^r &= Q \end{aligned}$$

3.2.3 Discussion

Solving rational agents' optimization problem generates a demand curve. Changes to the dividend growth rate instigate a parallel shift in the demand curve, and changes to risk aversion and volatility alter the slope of the demand curve. Rational agents hold all risky assets at the equilibrium price, which correlates positively with the dividend growth rate and negatively with risk aversion, the asset supply, and the volatility of the dividend. When the dividend growth rate is high, the asset price is also high, but when the risky asset is volatile, the asset price is low. Rational agents require compensation for bearing risk, and high risk aversion consequently drives down asset prices. The risky asset price is also affected by the riskless asset since the latter represents an alternative investment opportunity. Higher expected returns on the riskless asset alter rational agents' investment strategies and prevent them from being willing to hold the risky asset. To provide incentives, the risky asset price must decrease accordingly.

The risk-free rate is determined at time 0 by the central bank. At time 0, initial consumption is $C_0^r = rW_0^r - \frac{r-\delta}{r\gamma} + \frac{\gamma\sigma_D^2 Q^2}{2r}$. The central bank seeks to optimize utility at this time, which leads to a corollary.

Corollary 4 *Given the objective of the central bank, the optimal risk-free rate is:*

$$r = \sqrt{\frac{\frac{\gamma\sigma_D^2 Q^2}{2} + \frac{\delta}{\gamma}}{W_0^r}}$$

The optimal risk-free rate is determined by several economic variables, including investors' preferences, risky asset characteristics, and aggregated wealth in the economy. The central bank plays a role only at time 0 in helping the economy determine a risk-free rate, but does not participate in the general equilibrium after time 0. Investors thus do not consider the central bank's role in the economy, and the risk-free rate is not influenced by investors' strategies.

Primary issues related to asset pricing theory are the equity premium and excess volatility puzzles. The share equity premium is defined as the per-unit time expectation for the sum of the excess price changes and dividends, and the return equity premium is defined as the expectation for one dollar investment per unit of time less the risk-free rate, which is similar to the risk premium from Mehra and Prescott (1985). The return volatility is the diffusion term of the return process, which is expressed as the logarithm of price. These measures are expressed as a corollary:

Corollary 5 *In the economy, the share equity premium is:*

$$\frac{\mathbb{E}[(D_t - rP_t^r)dt + dP_t^r]}{dt} = \frac{\gamma Q \sigma_D^2}{r}$$

the return equity premium is:

$$\mathbb{E}[ER_t] = \frac{\gamma Q \sigma_D^2}{rP_t}$$

and the return volatility is:

$$Vol_{R_t}^r = \frac{\sigma_D}{rP_t}$$

In this model, factors that influence equity premium include the risk-free rate, uncertain dividends, risk aversion, and risky asset supplies. To solve the equity premium puzzle, some researchers impose additional structures, especially time-varying risk aversion (Campbell and Cochrane (1999)), recursive preferences (Bansal and Yaron (2004)), and production models (Choi and Mertens (2006); Hirshleifer et al. (2015)). Although these structures are compatible with our model, they are not the focus of the paper. For simplicity and tractability, these structures are reserved for future extensions.

We apply values from Barberis et al. (2015b) to obtain a numerical result. We also set the dividend growth rate to 5% and dividend volatility to 25% so that the Sharpe ratio matches real post-war data. Risk aversion γ is set to 0.1, and time discount rate δ is set to 1.5%. The supply of the risky asset is set to 5. We further set the initial wealth in the economy to 365. The central bank thus sets the optimal risk-free rate to 2.5%, which accords with the historical average in the data Mehra (2003). In this economy, the return equity premium is 0.96% and return volatility is 7.69%. Both numbers are much lower than the 7.8% annual excess return and 17% volatility in the real data.

3.3 The MAX-CAPM

This section introduces a heterogeneous-agent, consumption-based capital asset pricing model in a non-competitive market called MAX-CAPM. We assume that monopolistic traders, arbitrageurs, and extrapolators coexist. Extrapolators act as irrational agents with incorrect probability beliefs regarding risky asset prices. Instead of pursuing fundamental value, they create expectations of future price changes based on sentiment, which is modeled as a weighted average of past price changes. Arbitrageurs trade on price relative to fundamental values. Monopolistic traders are rational in the sense that they are aware of the true probability distribution of risky asset prices after incorporating the other two agents' strategies. They understand that their trades affect the equilibrium price, and they consider their market power. A similar three heterogenous agents model can be found in De Long et al. (1990b), who use a discrete-time, three-period context to study the strategies and performance of feedback traders, passive traders, and rational speculators. In their model, agents' strategies are specified exogenously, and feedback traders have very short memories regarding price changes. In our model, agents construct optimal strategies and extrapolators' memories decay slowly.

3.3.1 The Setup

Consider a continuous-time, infinite-horizon economy in which uncertainty is represented by a complete probability space of $(\Omega, \mathbb{F}, \mathbb{P}, \mathbb{P}^e)$ and information filtration of $(\mathcal{F}_t)_{t\geq 0}$, where $\mathbb{P}$ denotes the true probability measure and $\mathbb{P}^e$ extrapolators' incorrect probability. $(\mathcal{F}_t)_{t\geq 0}$ is the information set observed up to time t by extrapolators and monopolistic traders. Unlike in the information-based model in which agents use disparate information sets, in our model, extrapolators and the monopolistic traders share the same information set but process information differently. Agents form strategies based on probability beliefs of risky asset prices. For extrapolators, probability beliefs shift from the true probability distribution by a monotonic function of sentiment, the heterogeneity in beliefs that incurs trades.

3.3.2 Assets

Consider an economy with a single perishable consumption good that includes two assets:

1. A risky asset with a fixed supply that generates an instantaneous dividend rate of D_t and for which the market price is denoted by P_t. The dividend rate of D_t is governed by diffusion:

$$dD_t = g_D dt + \sigma_D dZ_t^D$$

where g_D is the deterministic dividend growth rate and σ_D is volatility. We assume they are both positive and constant. dZ_t^D is a Brownian motion under $\mathbb{P}$. The benchmark model shows that the fundamental price of this risky asset is determined by the dividend process. The dividend process of the risky asset is not observable directly. Instead, if agents have correct probability beliefs for the risky asset price, they are able to derive the correct dividend process. However, the dividend processes under $\mathbb{P}$ and $\mathbb{P}^e$ are different.

2. A riskless asset with a constant rate of return r ($r \geq 0$). At time 0, the central bank assumes a rational economy and sets the risk-free rate to maximize utility at this time. Risk-free rate r is given by corollary 4, and is constant thereafter.

3.3.3 Agents

There are three typical types of agents in the economy. The context of these three types of agents is supported by various finance literature and empirical evidence.

Arbitrageurs

The first type of agents, arbitrageurs, believes that current mispricing will be corrected in the long-run. They consequently trade against mispricing, defined as the difference between the observed market price and fundamental value. Arbitrageurs' demand for the risky asset is

$$dN_t^a = \alpha[(P_t^r - P_t) - N_t^a]dt + \sigma_a dZ_t^a$$

Arbitrageurs' demand is a mean-reverting Ornstein–Uhlenbeck process with long-run mean $P_t^r - P_t$, convergence rate α, and diffusion parameter σ_a. dZ_t^a is a Brownian motion under $\mathbb{P}$. The fundamental value is provided in the benchmark model with homogeneous rational agents, and market price is determined by general equilibrium, defined later.

Suppose that the market price is larger than the fundamental value so that there is a price bubble. Arbitrageurs believe that the bubble will burst in the long-run and tend to sell the risky asset. In this case, the long-run mean of their demand, $P_t^r - P_t$, is negative. The same logic applies to a negative mispricing. When there is no mispricing, such that the market price equals the fundamental value, arbitrageurs' target demand is zero. In this case, there is no arbitrage opportunity, and arbitrageurs exit the market. However, the long-run mean is changing dynamically. When there is no arbitrage opportunity, arbitrageurs seek to clear their positive position and exit the market. As they begin to sell, asset prices drop and a misprice is created, and a new arbitrage opportunity appears.

A large α, a liquidity indicator, indicates that the market is liquid for arbitrageurs. In this case, their demand converges to the long-run mean quickly. The inverse of the diffusion parameter, σ_a, measures information precision among arbitrageurs, and a large σ_a represents high noise in arbitrageurs' information set, which prevents their strategy from reverting to the long-run mean. Many studies investigate interactions between informed and uninformed investors, suggesting that information precision determines the equilibrium price. This paper incorporates this context in the sense that arbitrageurs serve as uninformed investors, while monopolistic traders serve as informed investors. Information precision indicates the difference in their information sets.

In some studies, arbitrageurs are strategic traders who consider counterparties' strategies. De Long et al. (1990a) assume that rational arbitrageurs are aware of noise traders. Shleifer and Vishny (1997) assume that arbitrageurs are institutional investors, and concern that pessimistic traders become even more pessimistic prevents arbitrageurs from taking large arbitrage positions. In these studies, risk aversion is relevant and arbitrageurs act as speculators. In contrast, our model considers arbitrageurs passive investors who have bounded rationality and consider only fundamental value. The same context can be found in De Long et al. (1990b), where passive investors' trading strategies depend only on price relative to fundamental value. De Long et al. (1990b) do not address why arbitrageurs cannot eliminate mispricing, but this paper argues for this issue from the limits of arbitrage, particularly from liquidity and information perspectives. Arbitrageurs are rational since they observe mispricing and correct it in the long-run. We endogenize arbitrageurs by having them incorporate the observed equilibrium price in their strategies.

Extrapolators

The second type of agents, extrapolators, incorrectly believes that risky asset prices are influenced by sentiment through channel:

$$dP_t^e = (\lambda_0 + \lambda_1 S_t)dt + \vec{\sigma}_P d\vec{Z}_t^e$$

where λ_0 and λ_1 determine the drift of extrapolators' conjectured risky asset price process. We assume $\lambda_0 \gtrsim 0$ because extrapolators are bullish generally (De Long et al. (1990a); Greenwood and Shleifer (2014)). λ_1 measures the influence of sentiment. In a market in which extrapolators are more sensitive to sentiment (e.g., an emerging market), λ_1 is expected to be large. $\vec{\sigma}_P$ is a two-dimensional vector of diffusion, and $d\vec{Z}_t^e$ is a two-dimensional Brownian motion under extrapolators' subjective probability measure $\mathbb{P}^e$. The functional form of diffusion term $\vec{\sigma}_P d\vec{Z}_t^e$ is specified in Appendix B.

$dP_t^e = P_{t+\Delta}^e - P_t$ represents extrapolators' conjectured risky asset price process, where P_t is the observed equilibrium price at time t, and $P_{t+\Delta}^e$ extrapolators' conjectured equilibrium price after a period of Δ. Therefore, in $\mathbb{P}^e$, the time t projection of time $t+\Delta$ risky asset price is normally distributed as

$$P^e_{t+\Delta}|P_t \sim N(P_t + (\lambda_0 + \lambda_1 S_t)\Delta, ||\vec{\sigma}_P \Delta||^2)$$

At time $t+\Delta$, extrapolators observe realized market price $P_{t+\Delta}$. They do not update their strategies since $P_{t+\Delta}$ is in the domain of the distribution above. They instead simply consider it a realization that is different from the expectation by normally distributed random noise.

We apply the same setting as in Barberis et al. (2015b) to model sentiment and extrapolators' beliefs. Sentiment S_t evolves according to diffusion:

$$dS_t = -\beta S_t dt + \beta dP_t$$

where $dS_t = S_{t+\Delta} - S_t$ is the change in sentiment, and $dP_t = P_{t+\Delta} - P_t$ the change in market price. It is an Ornstein–Uhlenbeck process with long-run mean zero that depends on price changes. The sentiment process is specified as mean-reverting because people tend to forget past events and enthusiasm fades. Price changes that occurred a month prior do not have the same effect on sentiment as price changes that occurred today. β measures the effect of price changes on sentiment. When β is large, sentiment is determined primarily by the most recent price change. In this case, extrapolators tend to forget past events more quickly. When β is low, even price changes in the distant past have a significant effect on current sentiments.

Extrapolators hold incorrect beliefs in subjective probability measure $\mathbb{P}^e$. At each time t that they conjecture, sentiment evolves according to:

$$dS_t = -\beta S_t dt + \beta dP^e_t$$

Extrapolators also believe that the dividend evolves according to:

$$dD_t^e = g_D^e(S_t, N_t^a)dt + \sigma_D d\hat{Z}_t^D$$

where their subjective dividend growth rate is specified in Lemma 1.

Lemma 2 *Dividend growth rate $g_D^e(S_t, N_t^a)$ in extrapolators' subjective probability measure is determined implicitly and depends on sentiment S_t and arbitrageurs' strategy N_t^a.*

In $\mathbb{P}^e$, arbitrageurs' demand process is:

$$d(N_t^a)^e = \alpha(f[g_D^e(S_t, N_t^a)] - N_t^a)dt + \sigma_a d\hat{Z}_t^a$$

which is also an Ornstein–Uhlenbeck process. α is a liquidity indicator, and σ_a measures information precision as defined in arbitrageurs' strategies. $\hat{Z}_t^a$ is a Brownian motion under $\mathbb{P}^e$. What arbitrageurs and extrapolators do not agree on is the long-run mean. In $\mathbb{P}^e$, long-run mean $f[g_D^e(S_t, N_t^a)]$ is an increasing function of dividend drift for extrapolators, which is also an increasing function of sentiment. For simplicity and without loss of generality, we assume $f[g_D^e(S_t, N_t^a)] = S_t$ throughout this paper. Therefore, extrapolators believe that arbitrageurs trade on sentiment instead of mispricing.

Monopolistic Traders

Monopolistic traders always hold correct beliefs about the price process and arbitrageurs' demand process. We derive monopolistic traders' strategies in both competitive and non-competitive markets, described below.

1. In a competitive market, monopolistic traders do not have market power. They are more likely to be called smart or knowledgeable traders.

2. In a non-competitive market, monopolistic traders have market power. They are aware that their strategies will have significant influence on asset prices. They incorporate this effect in their optimal strategy.

A typical example of a monopolistic trader is a large institutional investor with a knowledgeable and experienced research team that is able to investigate the behaviors of other market participants. They study sentiment formation among extrapolators and constraints that arbitrageurs experience. In the second context above, monopolistic traders also consider the price influence. In the literature, the roles of these three agents sometimes overlap. For example, Sias et al. (2006) assume that institutional investors are extrapolators, and Shleifer and Vishny (1997) that they are rational arbitrageurs. These contexts can be incorporated into our model but are left to future research.

3.3.4 Market Clearing

The market clears in equilibrium, and supply always equals demand. The market clearing condition is:

$$\mu_0 N_t^e + \mu_1 N_t^a + \mu_2 N_t^m = Q \tag{3.1}$$

where μ_0, μ_1, and $\mu_2 = 1-\mu_0-\mu_1$ specify the relative mass of each type of agent. The market clearing condition determines the equilibrium price endogenously. Given a fixed supply of risky assets, when total demand increases or decreases, P_t also increases or decreases. This condition therefore serves as a price impact function. A temporary or permanent price impact function is usually specified exogenously in large trader and market manipulation literature. This paper contributes to such literature by endogenizing the price impact function, which is embedded in the equilibrium model. We thus incorporate large traders into the equilibrium model without requiring additional assumptions.

3.3.5 Equilibrium

A monopolistic traders–arbitrageurs–extrapolators rational expectations equilibrium (MAX-REE) in an economy in which three heterogeneous agents coexist is defined as consumption and investment strategies $\{(C_t^e, N_t^e), (C_t^m, N_t^m)\}_{t\geq 0}$ and price process $\{P_t\}_{t\geq 0}$ such that:

1. Each agent chooses the consumption and investment strategy to maximize expected utility given probability beliefs $\{\mathbb{P}, \mathbb{P}^e\}$ and filtration $(\mathcal{F}_t)_{t\geq 0}$.
2. All agents share the same information set. Extrapolators hold incorrect beliefs, arbitrageurs trade against mispricing, and monopolistic traders hold both correct beliefs and market power.
3. Price P_t clears the security market.

3.3.6 Optimization Problem

Extrapolators

Extrapolators maximize expected CARA utility at time 0 given probability space $(\Omega, \mathbb{F}, \mathbb{P}^e)$ and information filtration $(\mathcal{F}_t)_{t\geq 0}$ as

$$\mathbb{E}_0^e\left[-\int_0^\infty \frac{e^{-\delta t-\gamma C_t^e}}{\gamma}dt\right]$$

subject to budget constraint and state variables processes:

$$\begin{aligned}
dW_t^e &= (rW_t^e - C_t^e)dt + N_t^e[(D_t - rP_t)dt + dP_t^e] \\
&= (rW_t^e - C_t^e)dt + N_t^e[(e_0^e + e_S^e S_t + e_{N^a}^e N_t^a)dt + \vec{\sigma}_P d\vec{Z}_t^e] \\
dS_t &= \beta(-S_t)dt + \beta(\lambda_0 + \lambda_1 S_t)dt + \beta\vec{\sigma}_P d\vec{Z}_t^e \\
&= \beta[\lambda_0 + (\lambda_1 - 1)S_t]dt + \beta\vec{\sigma}_P d\vec{Z}_t^e \\
dN_t^a &= \alpha(S_t - N_t^a)dt + \sigma_a d\hat{Z}_t^a
\end{aligned}$$

e_0^e, e_S^e, and $e_{N^a}^e$ are specified in Appendix B, and a superscript e is used to indicate extrapolators' probability measure of $\mathbb{P}^e$. $e_0^e + e_S^e S_t + e_{N^a}^e N_t^a$ thus measures the expected excess return for holding one share of risky asset under $\mathbb{P}^e$, where the dynamic of risky asset prices correlates positively with S_t. Sentiment increases when there is a positive price change, and the magnitude of the impact increases if the price change occurred recently. Extrapolators also believe that the long-run equilibrium of arbitrageurs' demand depends on sentiment; when sentiment is high, extrapolators believe that arbitrageurs are likely to increase their investments.

Monopolistic Traders and Competitive Market

In this case, monopolistic traders act like smart and knowledgeable traders in the sense that they hold correct beliefs in a competitive market. All three types of agents are price-takers. In this context, monopolistic traders are rational agents in the traditional asset pricing model. Barberis et al. (2015a) use a model without the presence of arbitrageurs to study interactions between extrapolators and rational agents. Monopolistic traders maximize expected CARA utility at time 0 given the correct probability space: $(\Omega, \mathbb{F}, \mathbb{P})$

and information filtration $(\mathcal{F}_t)_{t\geq 0}$ as

$$\mathbb{E}_0^m\left[-\int_0^\infty \frac{e^{-\delta t-\gamma C_t^m}}{\gamma}dt\right]$$

subject to budget constraint and state variables processes:

$$\begin{aligned}
dW_t^m &= (rW_t^m - C_t^m)dt + N_t^m[(D_t - rP_t)dt + dP_t] \\
&= (rW_t^m - C_t^m)dt + N_t^m[(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a)dt + \vec{\sigma}_P d\vec{Z}_t] \\
dS_t &= -\beta S_t dt + \beta dP_t \\
dN_t^a &= \alpha[(P_t^r - P_t) - N_t^a]dt + \sigma_a dZ_t^a
\end{aligned}$$

e_0^m, e_S^m, and $e_{N^m}^m$ are specified in Appendix B, and superscript m indicates monopolistic traders. $e_0^m + e_S^m S_t + e_{N^m}^m N_t^a$ measures the expected excess return for holding one share of risky asset under true measure $\mathbb{P}$. P_t is the true price process of the risky asset, and N_t^a is arbitrageurs' true demand. Monopolistic traders use the true price process to update expected sentiment, and they believe that arbitrageurs trade against mispricing. They are aware of the limits of arbitrage, so arbitrageurs cannot always eliminate mispricing.

Monopolistic Traders and Non-Competitive Market

In this case, in addition to superior knowledge about the financial market and other agents' strategies, monopolistic traders also have market power. They incorporate the endogenous price impact function into the optimal strategy. They maximize expected CARA utility at time 0 given correct probability space $(\Omega, \mathbb{F}, \mathbb{P})$ and information filtration $(\mathcal{F}_t)_{t\geq 0}$ as:

$$\mathbb{E}_0^m\left[-\int_0^\infty \frac{e^{-\delta t-\gamma C_t^m}}{\gamma}dt\right]$$

subject to budget constraint and state variables processes:

$$\begin{aligned}
dW_t^m &= (rW_t^m - C_t^m)dt + N_t^m[(D_t - rP_t)dt + dP_t] \\
&= (rW_t^m - C_t^m)dt + N_t^m[(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a + e_{N^m}^m N_t^m)dt + \vec{\sigma}_P d\vec{Z}_t] \\
dS_t &= -\beta S_t dt + \beta dP_t \\
dN_t^a &= \alpha[(P_t^r - P_t) - N_t^a]dt + \sigma_a dZ_t^a
\end{aligned}$$

e_0^m, e_S^m, $e_{N^a}^m$, and $e_{N^m}^m$ are specified in Appendix B. Results in section 5 show that they are different from the competitive MAX-CAPM. Additional term $e_{N^m}^m N_t^m$ measures the impact of monopolistic traders' investments on the instantaneous excess return of one share of risky asset. This term influences the equilibrium price through the optimization problem. Market power offers monopolistic traders a trade-off. Higher demand increases overall profit, but also drives up market prices and reduces per share expected return. Therefore, an optimal solution exists. $e_0^m + e_S^m S_t + e_{N^m}^m N_t^a + e_{N^m}^m N_t^m$ measures the expected excess return for holding one share of risky asset under true measure $\mathbb{P}$. P_t is the true price process of the risky asset, and N_t^a is arbitrageurs' true demand.

Monopolistic traders earn higher profit in non-competitive markets. To understand this phenomenon, consider a firm's profit maximization problem in standard economic theory. In a competitive market, a firm acts as a price-taker and its revenue is the product of exogenous unit price and quantity sold. In a non-competitive market, a firm acts as a monopolist and its revenue is the product of endogenous unit price and quantity sold, where the unit price is a function of quantity sold, derived from the consumer demand curve. Based on microeconomic theory, monopolists always earn higher revenue because of market power. The MAX-CAPM conveys similar logic; the expected excess return is a function of each agent's trading strategy, derived from the market clearing condition. $e_{N^m}^m N_t^m$ in the optimization problem reflects market power.

3.4 Equilibrium

In this section, we present equilibrium prices and optimal strategies in closed form for the MAX-CAPM. Other results, including mispricing, equity premium, and asset return volatility are also discussed.

3.4.1 Equilibrium Solutions

The model is developed in continuous-time. A dynamic programming approach (i.e., the HJB equation) is used to derive PDEs, which describe agents' value functions. A system of polynomial equations is used to determine the parameters of the model and obtain a proposition:

Proposition 5 *In MAX-CAPM, there exists a rational expectation equilibrium such that:*

- *The equilibrium risky asset price is:*

$$P_t = p_0 + \frac{D_t}{r} + p_S S_t + p_a N_t^a \tag{3.2}$$

- *Extrapolators' value function is:*

$$J_t^e = -e^{-\delta t - r\gamma W_t^e + a^e S_t^2 + b^e (N_t^a)^2 + c^e S_t N_t^a + d^e S_t + e^e N_t^a + f^e} \tag{3.3}$$

- *Monopolistic traders' value function is:*

$$J_t^m = -e^{-\delta t - r\gamma W_t^m + a^m S_t^2 + b^m (N_t^a)^2 + c^m S_t N_t^a + d^m S_t + e^m N_t^a + f^m} \tag{3.4}$$

$p_0, p_S, p_a, a^e, b^e, c^e, d^e, e^e, f^e, a^m, b^m, c^m, d^m, e^m, f^m$ are solutions to a system of fifteen polynomial equations specified by $\{$(B.9), (B.14), and (B.15)$\}$ in the competitive MAX-REE and $\{$(B.9), (B.22), and (B.23)$\}$ in the non-competitive MAX-REE.

The value functions are derived from each agent's continuous-time, dynamic programming problem. The idea is to rely on the HJB equation to obtain PDEs, and then use a system of polynomials to determine the parameters of conjectured solutions to the PDEs. According to the dynamic programming technique, applying Ito's formula to value functions produces drift-diffusion, and the optimal strategy optimizes each agent's value function so that the drift term equals zero in every state of the world. This condition allows for optimal strategies and is invariant with state variables, while each agent's optimal utility is determined based on state variables.

A set of parameters, including the discount rate, risk aversion, liquidity, and information precision, are used to describe the economy. Consistent with the example in the benchmark model, we set the dividend growth rate to 5% and dividend volatility to 25% so that the Sharpe ratio matches real post-war data. Risk aversion is set to 0.1 and time discount to 1.5%. The supply of the risky asset is set to 5. We assume that β equals 0.5 so that both the previous sentiment and the current price change contribute to 50% of the current sentiment. λ_0 is set to 0.1 so investors are slightly bullish generally. The liquidity indicator and information noise are normalized to one. Market shares of monopolistic traders, arbitrageurs, and extrapolators are 50%, 10%, and 40%, respectively. Discussions of the influence of these parameters appear in section 5.

The primary result of this paper is stated in Proposition 5. The equilibrium price is a linear function of three state variables—D_t, S_t, and N_t^a. Three parameters, p_0, p_S, and p_a, are determined jointly by the current economy. In comparison to the benchmark model, the equilibrium price is influenced by two more state variables—sentiment and arbitrageurs' current position. p_S describes the sensitivity of equilibrium prices to sentiment. Using real-world data to construct a sentiment measure, Baker and Wurgler (2006, 2007) show that high sentiment drives up current equilibrium prices and predicts low future returns. Our solution indicates that a positive p_S equals 0.74, which verifies their finding. p_a describes the sensitivity of equilibrium prices to arbitrageurs' current positions. Both monopolistic traders and extrapolators consider the arbitrageurs' role when constructing optimal strategies. However, since they hold disparate beliefs about future asset prices, their beliefs about arbitrageurs' strategies also differ. Monopolistic traders believe that arbitrageurs trade against mispricing, but extrapolators believe that they trade on sentiment. They agree that arbitrageurs are subject to the limits of arbitrage so that their demand process is mean-reverting. The convergence rate is a liquidity indicator. A more liquid market means that arbitrageurs' demand converges to a long-run mean more quickly. The diffusion term measures information noise. With more noise in the market, arbitrageurs' demand is more likely to deviate from their theoretical target.

Consistent with monopolistic traders' beliefs, arbitrageurs trade on mispricing, specified in a corollary:

Corollary 6 *Mispricing of asset prices is the difference between the market equilibrium and fundamental prices. Mathematically, mispricing, Δ_t, is expressed as:*

$$\begin{aligned}\Delta_t &= P_t - P_t^r \\ &= (p_0 - p_0^r) + p_S S_t + p_a N_t^a\end{aligned}$$

where

$$p_0^r = \frac{g_D - \gamma Q \sigma_D^2}{r^2}$$

Similar to the equilibrium price, our solution demonstrates that mispricing is an increasing function of sentiment and arbitrageurs' positions. $\Delta_t < 0$ indicates a negative asset price bubble and thus predicts high future returns. Extrapolators consider only sentiment, and monopolistic traders and arbitrageurs always consider the fundamental value and mispricing. Non-strategic arbitrageurs aim to correct mispricing, but strategic monopolistic traders aim to take advantage of their knowledge and exploit other agents. Monopolistic traders lower the current market price to generate high future returns, and market power enables them to strengthen their strategies. The solution shows that non-competitive MAX-CAPM generates a market price 27% lower than the fundamental price, and competitive MAX-CAPM generates a market price of only 12% lower than the fundamental price.

3.4.2 Dynamic of the Stochastic Process

Knowing that the equilibrium risky asset price is a function of state variables, the dynamic of the stochastic process can be expressed explicitly as:

Corollary 7 *The dynamics of the risky asset price evolve according to:*

$$dP_t = \frac{1}{1-p_S\beta}\left[\frac{g_D}{r} + p_a\alpha(p_0^r - p_0) - (p_S\beta + p_a p_S\alpha)S_t - p_a(p_a+1)\alpha N_t^a\right]dt + \vec{\sigma}_P d\vec{Z}_t$$

By introducing mispricing, the risky asset price process can also be expressed as:

$$dP_t = \frac{1}{1-p_S\beta}\left[\frac{g_D}{r} - p_a\alpha(N_t^a - \Delta_t) - p_S\beta S_t\right]dt + \vec{\sigma}_P d\vec{Z}_t \tag{3.5}$$

.

The risky asset price process is mean-reverting, with a long-run state equal to $\frac{g_D}{r}$. MAX-CAPM and the benchmark model agree on the long-run state, which depends only on the dividend growth rate and risk-free rate; it is invariant to investors' strategies. Since the dividend growth rate is constant and the risk-free rate is persistent after time 0, the long-run state is constant after time 0.

The presence of extrapolators and arbitrageurs adds two components to the drift. Sentiment S_t has a negative impact on risky asset price, and the impact increases when extrapolators are more sensitive to current price changes, or the equilibrium asset price is more sensitive to sentiment. In a bullish market in which extrapolators' sentiment is high, the asset price tends to decrease. Since arbitrageurs trade on mispricing, the difference between their current positions and mispricing also has a negative impact on the risky asset price, and the impact increases when arbitrageurs experience less liquidity constraints, or the equilibrium asset price is more sensitive to arbitrageurs' strategies. When there is a positive price bubble, the asset price tends to decrease. Intuitively, extrapolators are less likely to trade when they pay more attention to the entire price history, and arbitrageurs when they experience greater liquidity constraints. In this case, the risky asset price is more stable. The convergence rate correlates positively with extrapolators' sensitivity to the current price change and the sensitivity of the equilibrium price to sentiment:

Corollary 8 *The dynamics of sentiment and arbitrageurs' positions evolve according to:*

$$
\begin{aligned}
dS_t &= \frac{\beta}{1-p_S\beta}\left[\frac{g_D}{r} + p_a\alpha(p_0^r - p_0) - (1 + p_a p_S \alpha)S_t - p_a(p_a+1)\alpha N_t^a\right]dt \\
&\quad + \beta\vec{\sigma}_P d\vec{Z}_t \\
&= \frac{\beta}{1-p_S\beta}\left[\frac{g_D}{r} - p_a\alpha(N_t^a - \Delta_t) - S_t\right]dt + \beta\vec{\sigma}_P d\vec{Z}_t \qquad (3.6) \\
dN_t^a &= \alpha[(p_0^r - p_0) - p_S S_t - (p_a+1)N_t^a]dt + \sigma_a dZ_t^a \\
&= \alpha[-\Delta_t - N_t^a]dt + \sigma_a dZ_t^a
\end{aligned}
$$

The dynamic of the sentiment is mean-reverting, with a long-run state equal to $\frac{g_D}{r} - p_a\alpha(N_t^a - \Delta_t)$. Similar to the risky asset price process, the long-run state of sentiment depends on the dividend growth and risk-free rates. The long-run state changes dynamically with the difference between arbitrageurs' current position and mispricing, and it changes more rapidly with greater liquidity and more sensitive equilibrium prices.

In comparison to the dynamic of the risky asset price, the convergence rate of sentiment shrinks by a factor β, as does the volatility of sentiment. Sentiment is determined by both the current price change and price history. β measures the role of current price change during sentiment formation; the lower the β, the more slowly sentiment converges, and the less volatile sentiment is. In an extreme case in which β equals one, extrapolators consider only the current price change such that sentiment and the asset price move together. Since arbitrageurs trade against mispricing, the dynamic of arbitrageurs' positions is mean-reverting, with a long-run state equal to $-\Delta_t$. They experience liquidity constraints and information noise.

3.4.3 Volatility

Corollary 9 *In MAX-CAPM, the volatility of the risky asset price is:*

$$\vec{\sigma_P} = \sqrt{\frac{1}{(1-\beta p_S)^2}[(\frac{\sigma_D}{r})^2 + p_a^2\sigma_a^2]}$$

and the volatility of risky asset return at any time t is $\frac{\vec{\sigma_P}}{P_t}$*, where* P_t *is the equilibrium asset price.*

In comparison to the benchmark model, the volatility of risky asset price is significantly higher in the presence of extrapolators and arbitrageurs. Volatility consists of three components. $\frac{\sigma_D}{r}$ is the same as the volatility in the benchmark model; it is the contribution from fundamental risk. βp_s measures the contribution from extrapolators, and $p_a\sigma_a$ measures that from arbitrageurs. Recall that σ_a represents information noise, which causes arbitrageurs to trade unpredictably, thus adding variation in volatility. The variation is strengthened by the sensitivity of equilibrium price to arbitrageurs' current position.

The solution produces a return volatility of 17%, as opposed to 8% from the benchmark model. The 17% return matches the aggregated asset return volatility in the post-war S&P 500 data. MAX-CAPM thus solves the excess volatility puzzle. Other factors such as the economy and trading strategies affect volatility through p_s and p_a. However, the ability to solve the excess volatility puzzle remains. An extensive analysis appears in section 5.

3.4.4 Equity Premium

Returns under Extrapolators' Measure $\mathbb{P}^e$

Proposition 6 *The expected return of one share of a risky asset under extrapolators' measure $\mathbb{P}^e$ is:*

$$e_0^e + e_S^e S_t + e_{N_a}^e N_t^a$$

where $e_0^e = \lambda_0 - rp_0$, $e_S^e = \lambda_1 - rp_S$, $e_{N^a}^e = -rp_a$.

For extrapolators, the expected per-share return depends positively on the average bullish parameter, λ_0. The return is negatively sensitive to arbitrageurs' positions because it is mean-reverting. The sensitivity of the per-share return to sentiment can go either way. Extrapolators pay attention to the growth component and acknowledge that high sentiment drives asset prices up. However, they believe that the asset price continues to increase because of a multiplier of sentiment, which can be thought of as representing a value component. The join effect determines the sign and magnitude of expected per-share returns, such that the expected per-share return correlates positively to sentiment only when the value component is significant enough to compensate for the growth component. In this case, extrapolators' demand increases with high sentiment.

Returns under the Real Measure $\mathbb{P}$

Proposition 7 *Under competitive MAX-CAPM, monopolistic traders do not have market power. The expected return of one share of a risky asset under real measure $\mathbb{P}$ is:*

$$e_0^m + e_S^m S_t + e_{N_a}^m N_t^a$$

where

$$e_0^m = -rp_0 + \frac{g_D + \alpha r p_a(p_0^r - p_0)}{r(1 - p_S\beta)}$$
$$e_S^m = -rp_S - \frac{p_S\beta + p_a p_S \alpha}{1 - p_S\beta}$$
$$e_{N^a}^m = -rp_a - \frac{p_a(p_a + 1)\alpha}{1 - p_S\beta}$$

Under the real measure, results suggest that the sensitivity of per-share return to sentiment is negative, indicating that low sentiment, on average, predicts high future returns. The sensitivity of per-share return to arbitrageurs' positions is also negative, which accords with the supply–demand relationship.

Competitive MAX-CAPM results produce an excess return of 1.53%, which is more than the 0.96% excess return from the benchmark model. Using superior knowledge, monopolistic traders accumulate wealth at the loss of extrapolators. However, excess returns are still lower than the 7.8% return in post-war data. The equity premium puzzle remains a puzzle, and we therefore introduce a proposition:

Proposition 8 *Under non-competitive MAX-CAPM, monopolistic traders have market power. The expected return of one share of a risky asset under real measure $\mathbb{P}$ is:*

$$e_0^m + e_S^m S_t + e_{N_a}^m N_t^a + e_{N_m}^m N_t^m$$

where

$$e_0^m = \frac{r\gamma\vec{\sigma_P}^2 Q}{\mu_0} - (\lambda_0 + \beta d^e \vec{\sigma_P}^2 + e^e \frac{p_a\sigma_a^2}{1 - p_S\beta}) + \frac{g_D/r + p_a\alpha(p_0^r - p_0)}{(1 - p_S\beta)}$$
$$e_S^m = -(\lambda_1 + 2a^e\beta\vec{\sigma_P}^2 + c^e \frac{p_a\sigma_a^2}{1 - p_S\beta}) - \frac{p_S\beta + p_a p_S\alpha}{1 - p_S\beta}$$
$$e_{N^a}^m = -\frac{\mu_1 r\gamma\vec{\sigma_P}^2}{\mu_0} - (c^e\beta\vec{\sigma_P}^2 + 2b^e \frac{p_a\sigma_a^2}{1 - p_S\beta}) - \frac{p_a(p_a + 1)\alpha}{1 - p_S\beta}$$
$$e_{N^m}^m = -\frac{\mu_2 r\gamma\vec{\sigma_P}^2}{\mu_0}$$

Monopolistic traders experience a paradox. Increase in demand pushes the equilibrium price up. Since low asset prices generate high future returns, they want to hold less of the asset, but they also want to hold more to gain overall profit. Market power enables monopolistic traders to construct an optimal strategy. By introducing heterogeneous agents and market power, non-competitive MAX-CAPM produces an excess return of 7.51%, which matches real post-war data. In summary, non-competitive MAX-CAPM solves the equity premium puzzle and excess volatility puzzle simultaneously while maintaining a low and persistent risk-free rate, reasonable risk aversion, and return predictability.

3.4.5 Optimal Trading Strategies

Corollary 10 *Extrapolators' and the monopolistic traders' optimal demands are linear functions of state variables given by:*

$$N_t^e = f_0^e + f_S^e S_t + f_{N^a}^e N_t^a \tag{3.7}$$

$$N_t^m = f_0^m + f_S^m S_t + f_{N^a}^m N_t^a \tag{3.8}$$

This corollary shows the optimal trading strategy for each agent, given his or her probability belief. Extrapolators' trading strategies are linear functions of sentiment and arbitrageurs' current positions. Results suggest a positive coefficient for sentiment and a negative but negligible coefficient for arbitrageurs' current positions. When there is positive price change, extrapolators create bullish expectations of future price changes and speculate that arbitrageurs will follow. Monopolistic traders' strategies are also linear functions of sentiment and arbitrageurs' current positions. Results suggest negative coefficients for both sentiment and arbitrageurs' current positions, where sensitivity to arbitrageurs' current positions is much larger.

3.5 Numerical Analysis

MAX-CAPM ensures an analytic solution for asset prices by solving a system of polynomial equations. This section presents extensive analyses across various economies. The equilibrium asset price and its sensitivity to state variables are studied. Asset return, volatility, and trading strategies are also investigated. We also explore monopolistic traders' profitability to assess institutional investors.

The same parameters are chosen as in the previous section. The risk-free rate is set by the central bank at time 0 to maximize utility at that time. The dividend growth rate is 5%, the dividend volatility is 25%, risk aversion is 0.1, and the time discount factor is 1.5%. For extrapolators, β is set to 0.5, and λ_0 is set to 0.1 to indicate slightly bullish beliefs. The benchmark liquidity indicator and information precision are normalized to 1. In the benchmark context, market shares of monopolistic traders, arbitrageurs, and extrapolators are 50%, 10%, and 40%, respectively.

3.5.1 Limits of Arbitrage

Arbitrageurs trade against mispricing as:

$$dN_t^a = \alpha[-\Delta_t - N_t^a]dt + \sigma_a dZ_t^a$$

where Δ_t represents mispricing at time t. The limits of arbitrage prevent arbitrageurs from holding the target amount. This section investigates the limits of arbitrage from two aspects—liquidity and information. α measures the convergence rate and serves as a liquidity indicator, and diffusion parameter σ_a measures information noise, or the inverse of information precision.

Liquidity

The liquidity experiences by arbitrageurs is measured by α. In empirical asset pricing literature, several indicators are used to measure liquidity. For market liquidity, common measures include effective bid–ask spreads, market depth, price resilience, and a illiquidity measure proposed by Amihud (2002). For funding liquidity, common measures are TED spread and Libor-OIS spread. Hou et al. (2015) and Goyenko et al. (2009) offer a comprehensive review of liquidity measures. We treat market and funding liquidity as unified liquidity, and α measures the general liquidity condition. This context is supported by Brunnermeier and Pedersen (2008), who show that market and funding liquidity interconnect, and that there is a spiral effect between the two. A larger α means adequate liquidity for arbitrageurs to trade, and a smaller α means arbitrageurs' strategies converge to the long-run mean slowly. The liquidity indicator varies from 0.2 to 4 to demonstrate the evolution of results from an illiquid to liquid market.

Figure B.1 shows excess returns in markets with different liquidity. In non-competitive MAX-CAPM, the excess return falls between 7.5% and 8%, which aligns with post-war data. Additional analyses show that the volatility is between 16% and 19%. Therefore, varying the liquidity condition does not prevent MAX-CAPM from solving empirical puzzles. In competitive MAX-CAPM, the excess return is more stable at 1.5%, and volatility is below 12%, both of which are much lower than post-war data.

Figure B.2 shows results for the equilibrium price. Since low prices predict high future returns, monopolistic traders use their superior knowledge and market power to lower asset prices. The difference between price and market prices becomes larger in illiquid markets since arbitrageurs are unable to correct mispricing over time. In a competitive market, the market price is 13% lower than the fundamental price, whereas in a non-competitive market, market price is as much as 30% lower.

Figure B.3 shows that the expected profit collected by monopolistic traders over one unit of time increases strictly with liquidity. Profit is the product of optimal demand and per-share return. Knowing that arbitrageurs trade against mispricing, monopolistic traders' optimal demand increases in the same direction as per-share return, and market power enables them to gain three times more profit.

Information Precision

A common topic in finance literature is the information-based asset pricing model. In an economy with informed and uninformed investors, Wang (1993) shows that information precision is a state variable that affects equilibrium price. Epstein and Schneider (2008) find that investors dislike assets for which information quality is poor, especially when underlying fundamentals are volatile, and Kelly and Ljungqvist (2012) show that prices and uninformed demand decrease as information asymmetry increases. These decreases are larger when more investors are uninformed, turnover is larger and more variable, payoffs are more uncertain, and the lose signal is more precise.

Information precision is measured as the inverse of diffusion in arbitrageurs' strategies. Diffusion indicates the degree of information noise, which in this study increases gradually from 0.2 to 4 such that information precision experienced by arbitrageurs decreases and information asymmetry increases. Results appear in Figures B.4, B.5, and B.6.

The previous section shows that holding everything else constant, information noise correlates positively with the volatility of risky asset prices. Results demonstrate that without holding other parameters constant, increasing information noise still increases volatility of risky asset prices. The excess return increases with more information noise since it introduces additional risk, as shown in Figure B.4. The excess return under non-competitive MAX-CAPM is 7.50%, and under competitive MAX-CAPM is 1.50%. As more information noise is added, Figure B.5 shows that the equilibrium price decreases correspondingly, which corroborates findings from Kelly and Ljungqvist (2012).

A more liquid market and more information precision indicate an efficient market; Figures B.2 and B.5 show that they both increase equilibrium price. Shleifer and Vishny (1997) argue that arbitrageurs' information noise represents systematic risk that should be priced because it lowers the equilibrium price. Covariance between equilibrium price and information noise can be determined using the intertemporal capital asset pricing model or arbitrage pricing theory. These efficient market approaches assume that arbitrageurs see arbitrage opportunities and take them without constraint. Liquidity constraints, such as concern for being liquidated, prevent arbitrageurs from keeping desired demands, and lower the equilibrium price. The effect is stronger under non-competitive MAX-CAPM.

3.5.2 Market Share

Extrapolators invest according to sentiment, but arbitrageurs invest based on mispricing. The effect of market power depends not only on the economy and trading strategies, but on relative market shares. An investor should consider relative market shares when making investment decisions. One example is shifts between passive and active management. Active management dominated the market and brought in large amounts of excess returns during the first decade of the twenty-first century. As its market share increased, returns decreased and investors switched to passive management. Change in relative market shares again shifted returns and passive management gradually became crowded, allowing active investments to outpace passive investments during 2017[2]. Changes to relative market shares can shift the performance of disparate strategies. In this subsection, monopolistic traders' expected returns and profit are investigated based on investors' varying market shares.

Market Share: Extrapolators

Since 2013, households directly own 38% of the U.S. equity market[3], and a significant portion of them are extrapolators. In this subsection, we fix the market shares of arbitrageurs and vary relative market shares of extrapolators and monopolistic traders. The market share of extrapolators is approximately 40%.

Figure B.7 shows positive sensitivity of equilibrium prices to sentiment. Increasing the market shares of extrapolators leads more investors to form trading strategies based on sentiment, increasing the influence of sentiment on equilibrium pricing. Figure

[2]https://www.investopedia.com/news/active-vs-passive-investing

[3]http://www.businessinsider.com/chart-stock-market-ownership-2013-3

B.8 shows that the sensitivity of equilibrium prices to arbitrageurs' position changes as extrapolators' market shares increase. Under non-competitive MAX-CAPM, the equilibrium price decreases slightly with arbitrageurs' current positions, and under competitive MAX-CAPM, it increases significantly. Both extrapolators and monopolistic traders evaluate arbitrageurs' strategies; extrapolators believe that arbitrageurs trade on sentiment, and monopolistic traders believe they trade on mispricing. Market power strengthens the influences of beliefs.

Extrapolators drive price deviations and destabilize the market, and the volatility of risky asset returns increase as extrapolators' market shares increase. Theoretically, high volatility lowers current prices and generates higher expected returns and profit. Results suggest that this is true only in a competitive market. In a non-competitive market, as market shares of monopolistic traders decrease and more concentrated market power brings higher returns, each monopolistic trader is willing to hold more risky assets. This incentive offsets influences from increases to risk and drives current prices up, while expected excess returns decrease correspondingly. Since monopolistic traders' optimal demand increases while their expected returns decrease, their expected profit is convex in extrapolators' market shares and reaches a local minimum when extrapolators' shares are approximately 41%.

Market Share: Arbitrageurs

In this subsection, extrapolators' market shares are fixed, and arbitrageurs' and monopolistic traders' relative market shares vary. Figure B.13 and B.14 show the sensitivity of equilibrium prices to state variables. The sensitivity to arbitrageurs' positions increases with arbitrageurs' market shares since higher participation leads to more influence of the equilibrium price. The direction of sensitivity to sentiment differs between competitive and non-competitive markets. Generally, sentiment generates a high equilibrium price, and extrapolators' optimal demand under $\mathbb{P}^e$ deviates from optimal demand under $\mathbb{P}$ because of incorrect beliefs. Knowing the negative relationship between sentiment and future returns, monopolistic traders profit by exploring extrapolators' strategies, and market power amplifies the consequences.

Arbitrageurs mitigate mispricing and stabilize the market. With more arbitrageurs correcting mispricing, both extrapolators' and monopolistic traders' optimal demands and the volatility of risky asset return decrease. Low volatility leads to low expected returns and high equilibrium prices. Monopolistic traders' profit depends on the gap between arbitrageurs' current and target positions. If arbitrageurs have a strong tendency to buy or sell risky assets, each monopolistic trader's expected profit increases or decreases with arbitrageurs' higher market shares.

Among empirical finance studies on institutional ownership, Jones et al. (1999), Nofsinger and Sias (1999), Wermers (1999), Bennett et al. (2003), and Parrino et al. (2003) support the idea that increases to institutional ownership predicts asset returns positively. They define institutions as investors (1) that are informed and have correct beliefs about the distribution of future asset returns, and (2) whose trades have direct effects on asset prices. The definition is analogous to monopolistic traders in our model, and analyses in this subsection generate corroborating evidence. Under non-competitive MAX-CAPM, expected returns increase as monopolistic traders' market shares increase. Therefore, the model provides a theoretical foundation for empirical research on institutional ownership.

3.5.3 State Variables

In this subsection, the economic context is reset to benchmark values. We show that state variables influence not only equilibrium price, but also expected excess returns, and generate a ”profit smile.”

Sentiment

We gradually increase sentiment. Figures B.19, B.20, and B.21 show results. The equilibrium price increases with sentiment since the sensitivity of equilibrium price to sentiment is strictly positive. Market power increases the influence of state variables. The sensitivity to sentiment is 0.74 under non-competitive MAX-CAPM and 0.46 under competitive MAX-CAPM, and sensitivity to arbitrageurs’ current positions is 0.14 under non-competitive MAX-CAPM and 0.05 under competitive MAX-CAPM.

Baker and Wurgler (2006, 2007); Huang et al. (2015) demonstrate that high sentiment predicts low future returns. Results in Figure B.19 corroborate this finding. In our model, high sentiment provides incentives for extrapolators to increase their demand, which drives equilibrium prices up. With increasing sentiment, expected excess returns decrease from 4.39% to -6.59% under competitive MAX-CAPM and more significantly decrease from 14.92% to -13.95% under non-competitive MAX-CAPM .

Monopolistic traders are aware of the negative relationship between sentiment and expected returns, and they construct trading strategies accordingly. When sentiment is low, they buy risky assets, and when sentiment is high, they sell short instead. This strategy generates huge profits at the cost of extrapolators’ losses, especially under extreme sentiment scenarios. Figure B.21 shows a ”profit smile,” such that profit is non-negative and convex on sentiment. In a special case when arbitrageurs correct mispricing caused by sentiment, monopolistic traders collect zero profit.

Arbitrageurs' Positions

We gradually increase arbitrageurs' positions. Figures B.22, B.23, and B.24 show results. With an increase to arbitrageurs' positions, expected excess returns decrease from 3.16% to −1.91% under competitive MAX-CAPM and decrease more significantly from 13.09% to −6.93% under non-competitive MAX-CAPM. Figure B.24 shows a "profit smile," such that monopolistic traders' profit is non-negative and convex on arbitrageurs' current position. In a special case when sentiment is zero and arbitrageurs hold all risky assets, monopolistic traders collect zero profit.

3.6 Conclusion

Applying a consumption asset pricing model in a continuous-time environment, this paper investigates the roles of three heterogeneous agents in financial market—monopolistic traders, arbitrageurs, and extrapolators. A closed-form solution for equilibrium price and optimal strategy is derived. Parameters of the solution are obtained by solving a system of polynomials. By introducing a central bank, we endogenize the risk-free rate. Non-competitive MAX-CAPM simultaneously reproduces many characteristics of asset pricing while maintaining a low and persistent risk-free rate, reasonable risk aversion, and return predictability. Market power enables monopolistic traders to generate high expected excess returns, which accords with the equity premium puzzle. The volatility of asset return is strengthened by sentiment and the limits of arbitrage, which explains the excess volatility puzzle. Both results match real post-war data in the U.S. equity market.

This paper assesses a general equilibrium model with heterogeneous investors in a non-competitive market. It is the first non-competitive market general equilibrium asset pricing model that endogenizes the price impact. The idea that the market clearing condition serves as a price impact function sheds light on future research on this topic. The model covers most types of players in the financial market, including irrational and rational, informed and uninformed, passive and active, and individual and institutional investors. The model proposes hypotheses and provides theoretical foundations for several influential empirical asset pricing studies. For future research, it would be interesting to explore interactions between assumptions and derive profitable investment strategies. For example, the predictive power of sentiment under various institutional ownership markets and liquidity conditions should be analyzed. Fama-French empirical techniques should also be applied in this case.

APPENDIX A

APPENDIX OF CHAPTER 2

A.1 Arbitrageurs and Extrapolators' Problems

Consider an economy with the time structure and asset structure described in section 2. At each time t, the arbitrageurs and extrapolators maximize a CARA utility on next period wealth given by:

$$\max_{N_t^i} \mathbb{E}_t^i \left[-e^{-\gamma(W_t + N_t^i(\tilde{P}_{t+1} - P_t))} \right]$$

By assumption, although each agent has different beliefs about the expectation of $\tilde{P}_{t+1}$, it follows a normal distribution with variance equals to σ_ϵ^2. So the expression inside the expectation follows a log-normal distribution. Taking first order derivative with respect to N_t^i yields

$$N_t^i = \frac{\mathbb{E}_t^i(\tilde{P}_{t+1}) - P_t}{\gamma \sigma_\epsilon^2}$$

Arbitrageurs believe the asset price will return to fundamental value in the next period, whose expectation equals to the current realized fundamental value due to the structure of innovations. On the other hand, extrapolators believe the expected price change of asset price is a weighted average of past price changes. They form their expectations as follows

$$\begin{aligned}
\mathbb{E}_t^a(\tilde{P}_{t+1} - P_t) &= P_0 + \sum_{k=1}^{t} \epsilon_k - P_t \\
\mathbb{E}_t^e(\tilde{P}_{t+1} - P_t) &= (1-\theta) \sum_{k=1}^{t-1} \theta^{k-1}(P_{t-k} - P_{t-k-1}) + \theta^{t-1} X_1
\end{aligned}$$

The results obtain by pluging in these expressions. ■

A.2 Proofs

A.2.1 Proof of Lemma 1

We start from the price impact function at time $k-1$ and time $k-2$, where $k = 2, 3, ..., t$

$$P_{k-1} = P^r_{k-1} - \frac{\gamma\sigma^2_\epsilon Q}{\mu^a} + \frac{\mu^e}{\mu^a}X^e_{k-1} + \frac{\mu^m\gamma\sigma^2_\epsilon}{\mu^a}N^m_{k-1} \tag{A.1}$$

$$P_{k-2} = P^r_{k-2} - \frac{\gamma\sigma^2_\epsilon Q}{\mu^a} + \frac{\mu^e}{\mu^a}X^e_{k-2} + \frac{\mu^m\gamma\sigma^2_\epsilon}{\mu^a}N^m_{k-2} \tag{A.2}$$

The extrapolators' sentiment is a weighted average of previous price changes and previous sentiment, this recursive relation gives us an expression for X^e_k

$$\begin{aligned} X^e_k &= (1-\theta)(P_{k-1} - P_{k-2}) + \theta X^e_{k-1} \\ &= [(1-\theta)\frac{\mu^e}{\mu^a} + \theta]X^e_{k-1} - (1-\theta)\frac{\mu^e}{\mu^a}X^e_{k-2} + (1-\theta)[\epsilon_{k-1} + \frac{\mu^m\gamma\sigma^2_\epsilon}{\mu^a}(N^m_{k-1} - N^m_{k-2})] \end{aligned} \tag{A.3}$$

After some algebra we have

$$X^e_k - aX^e_{k-1} = b(X^e_{k-1} - aX^e_{k-2}) + Z_{k-1}$$

where

$$\begin{cases} Z_{k-1} = (1-\theta)[\epsilon_{k-1} + \frac{\mu^m\gamma\sigma^2_\epsilon}{\mu^a}(N^m_{k-1} - N^m_{k-2})] \\ a + b = (1-\theta)\frac{\mu^e}{\mu^a} + \theta \\ ab = (1-\theta)\frac{\mu^e}{\mu^a} \end{cases}$$

Let $Y_k = X^e_k - aX^e_{k-1}$, so that $Y_1 = X^e_1 - aX^e_0$ is given initially. Adding up the sequences we have

$$Y_k = b^{k-1}Y_1 + \sum_{i=1}^{k-1} b^{i-1}Z_{k-i}$$

replace Y_k by X_k^e

$$X_k^e = aX_{k-1}^e + b^{k-1}Y_1 + \sum_{i=1}^{k-1} b^{i-1}Z_{k-i}$$

Again, adding up the sequences we have

$$X_k^e = \sum_{j=1}^{k-1} a^{j-1}b^{k-j}Y_1 + \sum_{j=1}^{k-1}\sum_{i=1}^{k-j} a^{j-1}b^{i-1}Z_{k+1-j-i} + a^{k-1}X_1^e$$

After some algebra, X_k^e can be expressed as follows.

$$\begin{aligned} X_k^e = & \frac{a^k - b^k}{a-b}X_1^e - ab\frac{a^{k-1} - b^{k-1}}{a-b}X_0^e \\ & + (1-\theta)\eta \sum_{i=1}^{k-1} \frac{a^i - b^i}{a-b}(N_{k-i}^m - N_{k-i-1}^m) \\ & + (1-\theta) \sum_{i=1}^{k-1} \frac{a^i - b^i}{a-b}\epsilon_{k-i} \end{aligned}$$

where $\eta = \frac{\mu^m \gamma \sigma_\epsilon^2}{\mu^a}$. This gives us the result. ■

A.2.2 Proof of Proposition 1

The price impact functions are obtained by plugging sentiment into the market clearing conditions.

$$
\begin{aligned}
P_t &= P_t^r - \frac{\gamma\sigma_\epsilon^2 Q}{\mu^a} + \frac{\mu^e}{\mu^a}X_t^e + \frac{\mu^m\gamma\sigma_\epsilon^2}{\mu^a}N_t^m \\
&= P_t^r - \frac{\gamma\sigma_\epsilon^2 Q}{\mu^a} + \frac{\mu^e}{\mu^a}\left[\frac{a^t - b^t}{a-b}X_1^e - ab\frac{a^{t-1} - b^{t-1}}{a-b}X_0^e\right] \\
&+ (1-\theta)\eta\frac{\mu^e}{\mu^a}\sum_{i=1}^{t-1}\frac{a^i - b^i}{a-b}(N_{t-i}^m - N_{t-i-1}^m) + \eta N_t^m \\
&+ (1-\theta)\frac{\mu^e}{\mu^a}\sum_{i=1}^{t-1}\frac{a^i - b^i}{a-b}\epsilon_{t-i}
\end{aligned}
$$

This completes the proof.■

A.2.3 Proof of Proposition 2

Since the fundamental shock ϵ_i follows a normal distribution, monopolistic traders' time 1 utility follows a log-normal distribution. The expected utility can be transformed into a mean-variance utility over the liquidation value as follows.

$$
\max_{\{N_k^m\}_{(1\le k\le T)}} \mathbb{E}_1^m\left[f(\boldsymbol{N_1^m})\right] - \frac{1}{2}\gamma Var\left[f(\boldsymbol{N_1^m})\right]
$$

where

$$
f(\boldsymbol{N_1^m}) = (P_T - P_{T-1})N_{T-1}^m + \sum_{i=1}^{T-2}(P_{i+1} - P_i)N_i^m + mP_1
$$

Applying the results in proposition 1 gives the expression in terms of monopolistic traders' trading history.

$$
mP_1 = m(P_0^r - \frac{\gamma\sigma_\epsilon^2 Q}{\mu^a} + \frac{\mu^e}{\mu^a}X_1^e) + m\epsilon_1 + m\eta N_1^m
$$

$$
\begin{aligned}
(P_{i+1} - P_i)N_i^m = & \frac{\mu^e}{\mu^a}(\frac{a^{i+1} - b^{i+1}}{a-b} - \frac{a^i - b^i}{a-b})X_1^e N_i^m \\
& - \frac{\mu^e}{\mu^a}ab(\frac{a^i - b^i}{a-b} - \frac{a^{i-1} - b^{i-1}}{a-b})X_0^e N_i^m \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{k=1}^{i}(\frac{a^{i-k+1} - b^{i-k+1}}{a-b} - \frac{a^{i-k} - b^{i-k}}{a-b})(N_k^m - N_{k-1}^m)N_i^m \\
& + \eta N_i^m(N_{i+1}^m - N_i^m) \\
& + \epsilon_{i+1}N_i^m + \frac{\mu^e}{\mu^a}(1-\theta)\sum_{k=1}^{i}(\frac{a^{i-k+1} - b^{i-k+1}}{a-b} - \frac{a^{i-k} - b^{i-k}}{a-b})\epsilon_k N_i^m
\end{aligned}
$$

$$
\begin{aligned}
(P_T - P_{T-1})N_{T-1}^m = & \frac{\mu^e}{\mu^a}(\frac{a^T - b^T}{a-b} - \frac{a^{T-1} - b^{T-1}}{a-b})X_1^e N_{T-1}^m \\
& - \frac{\mu^e}{\mu^a}ab(\frac{a^{T-1} - b^{T-1}}{a-b} - \frac{a^{T-2} - b^{T-2}}{a-b})X_0^e N_{T-1}^m \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{k=1}^{T-1}(\frac{a^{T-k} - b^{T-k}}{a-b} - \frac{a^{T-k-1} - b^{T-k-1}}{a-b})(N_k^m - N_{k-1}^m)N_{T-1}^m \\
& - \eta(N_{T-1}^m)^2 \\
& + \epsilon_T N_{T-1}^m + \frac{\mu^e}{\mu^a}(1-\theta)\sum_{k=1}^{T-1}(\frac{a^{T-k} - b^{T-k}}{a-b} - \frac{a^{T-k-1} - b^{T-k-1}}{a-b})\epsilon_k N_{T-1}^m
\end{aligned}
$$

First order condition with respect to N_1^m yields:

$$
\begin{aligned}
0 = & [\frac{\mu^e}{\mu^a}(1-\theta) - 2]\eta N_1^m + \eta N_2^m \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{i=1}^{T-1}(\frac{a^i - b^i}{a-b} - \frac{a^{i-1} - b^{i-1}}{a-b})N_i^m \\
& - \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{i=2}^{T-1}(\frac{a^{i-1} - b^{i-1}}{a-b} - \frac{a^{i-2} - b^{i-2}}{a-b})N_i^m \\
& + \frac{\mu^e}{\mu^a}(a+b-1)X_1^e - \frac{\mu^e}{\mu^a}abX_0^e + [1 - \frac{\mu^e}{\mu^a}(1-\theta)]m\eta \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\epsilon_1 \\
& - \gamma\sigma_\epsilon^2 N_1^m
\end{aligned}
\tag{A.4}
$$

First order condition with respect to N_j^m where $(1 < j < T-1)$ yields:

$$
\begin{aligned}
0 = {} & \eta(N_{j+1}^m - 2N_j^m + N_{j-1}^m) \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{i=1}^{j}\left(\frac{a^{j-k+1} - b^{j-k+1}}{a-b} - \frac{a^{j-k} - b^{j-k}}{a-b}\right)(N_k^m - N_{k-1}^m) \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{i=j}^{T-1}\left(\frac{a^{i-j+1} - b^{i-j+1}}{a-b} - \frac{a^{i-j} - b^{i-j}}{a-b}\right)N_j^m \\
& - \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{i=j+1}^{T-1}\left(\frac{a^{i-j} - b^{i-j}}{a-b} - \frac{a^{i-j-1} - b^{i-j-1}}{a-b}\right)N_j^m \\
& + \frac{\mu^e}{\mu^a}\left(\frac{a^{j+1} - b^{j+1}}{a-b} - \frac{a^{j} - b^{j}}{a-b}\right)X_1^e - \frac{\mu^e}{\mu^a}ab\left(\frac{a^{j} - b^{j}}{a-b} - \frac{a^{j-1} - b^{j-1}}{a-b}\right)X_0^e \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\left(\frac{a^{j} - b^{j}}{a-b} - \frac{a^{j-1} - b^{j-1}}{a-b}\right)\epsilon_1 \\
& - \gamma\sigma_\epsilon^2 N_j^m - \gamma\sum_{k=2}^{j}\left(\frac{a^{j-k+1} - b^{j-k+1}}{a-b} - \frac{a^{j-k} - b^{j-k}}{a-b}\right)^2\left[\frac{\mu^e}{\mu^a}(1-\theta)\right]^2\sigma_\epsilon^2 N_j^m
\end{aligned}
\tag{A.5}
$$

First order condition with respect to N_{T-1}^m yields:

$$
\begin{aligned}
0 = {} & \eta(-2N_{T-1}^m + N_{T-2}^m) + \frac{\mu^e}{\mu^a}(1-\theta)\eta N_{T-1}^m \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\eta \sum_{k=1}^{T-1}\left(\frac{a^{T-k} - b^{T-k}}{a-b} - \frac{a^{T-k-1} - b^{T-k-1}}{a-b}\right)(N_k^m - N_{k-1}^m) \\
& + \frac{\mu^e}{\mu^a}\left(\frac{a^{T} - b^{T}}{a-b} - \frac{a^{T-1} - b^{T-1}}{a-b}\right)X_1^e - \frac{\mu^e}{\mu^a}ab\left(\frac{a^{T-1} - b^{T-1}}{a-b} - \frac{a^{T-2} - b^{T-2}}{a-b}\right)X_0^e \\
& + \frac{\mu^e}{\mu^a}(1-\theta)\left(\frac{a^{T-1} - b^{T-1}}{a-b} - \frac{a^{T-2} - b^{T-2}}{a-b}\right)\epsilon_1 \\
& - \gamma\sigma_\epsilon^2 N_{T-1}^m - \gamma\sum_{k=2}^{T-1}\left(\frac{a^{T-k} - b^{T-k}}{a-b} - \frac{a^{T-k-1} - b^{T-k-1}}{a-b}\right)^2\left[\frac{\mu^e}{\mu^a}(1-\theta)\right]^2\sigma_\epsilon^2 N_{T-1}^m
\end{aligned}
\tag{A.6}
$$

Equations A.4 - A.6 are $T-1$ linear equations with $T-1$ unknowns. Rearranging equations A.4 - A.6 gives us the expression for $\boldsymbol{A_1}$ and $\boldsymbol{b}$. ■

A.2.4 Proof of Corollary 1

The objective function $f(\boldsymbol{N_1^m})$ is a second order polynomial. If $f(\boldsymbol{N_1^m})$ is a concave function with respect to unknowns, it will have unique optimal solution. Therefore, the coefficient in front of each squared unknown has to be non-positive. Taking derivatives for polynomial doesn't change the sign of coefficients. The derivatives of each squared unknown are the diagonal elements in $\boldsymbol{A_1}$, which should be non-positive to ensure an optimal solution. ■

A.2.5 Proof of Proposition 3

From the results in proposition 2, setting $N_0^m = 0$ yields the results. ■

A.2.6 Proof of Corollary 3

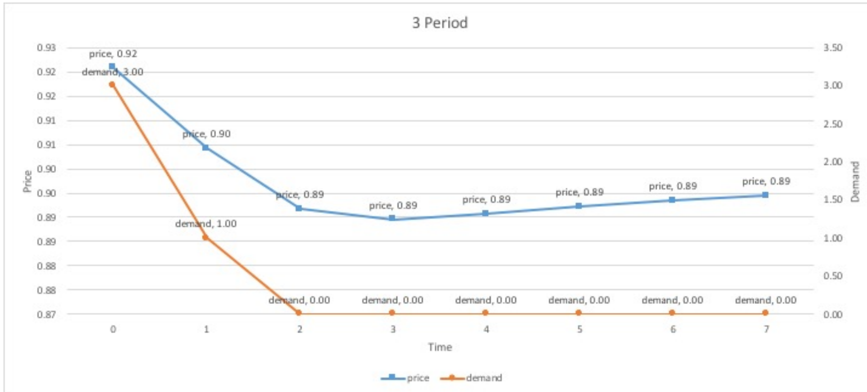
The market manipulators' optimal trading strategy $\boldsymbol{N_1^m} = \{N_1^m, ..., N_T^m\}_1$ at time 1 is $\boldsymbol{N_1^m} = \boldsymbol{X_1^{-1}} \boldsymbol{y_1}$. Where

$$\boldsymbol{y_1} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{T-1} \end{bmatrix}$$

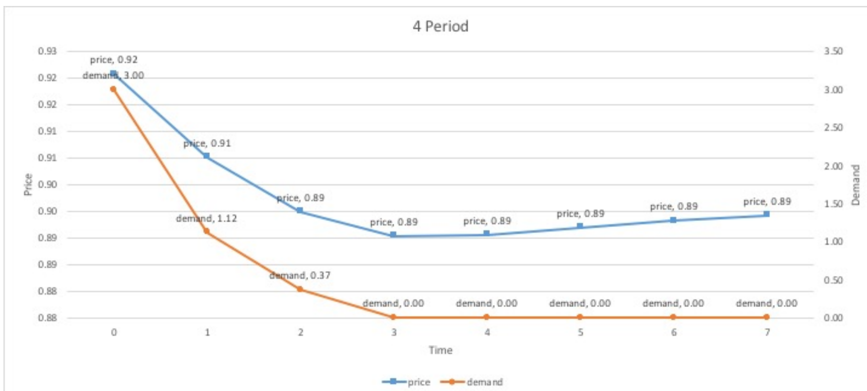
$$
\begin{aligned}
y_1 = & -\frac{\mu^e}{\mu^a}(a+b-1)X_1^e + \frac{\mu^e}{\mu^a}abX_0^e - [1-\frac{\mu^e}{\mu^a}(1-\theta)]m\eta - \frac{\mu^e}{\mu^a}(1-\theta)\epsilon_1 \\
y_j = & -\frac{\mu^e}{\mu^a}(\frac{a^{j+1}-b^{j+1}}{a-b} - \frac{a^j-b^j}{a-b})X_1^e + \frac{\mu^e}{\mu^a}ab(\frac{a^j-b^j}{a-b} - \frac{a^{j-1}-b^{j-1}}{a-b})X_0^e \\
& -\frac{\mu^e}{\mu^a}(1-\theta)(\frac{a^j-b^j}{a-b} - \frac{a^{j-1}-b^{j-1}}{a-b})\epsilon_1 \\
y_{T-1} = & -\frac{\mu^e}{\mu^a}(\frac{a^T-b^T}{a-b} - \frac{a^{T-1}-b^{T-1}}{a-b})X_1^e + \frac{\mu^e}{\mu^a}ab(\frac{a^{T-1}-b^{T-1}}{a-b} - \frac{a^{T-2}-b^{T-2}}{a-b})X_0^e \\
& -\frac{\mu^e}{\mu^a}(1-\theta)(\frac{a^{T-1}-b^{T-1}}{a-b} - \frac{a^{T-2}-b^{T-2}}{a-b})\epsilon_1
\end{aligned}
$$

The initial position m equals to zero by the nature of market manipulator. Fundamental shocks are ϵ_i. Initial sentiments are X_0^e and X_1^e. In an economy without fundamental shock and with zero initial sentiment we have $\boldsymbol{y_1} = \mathbf{0}$, so that $\boldsymbol{N_1^m} = \mathbf{0}$ as well. The monopolistic traders' optimal strategy is to not to trade. ■

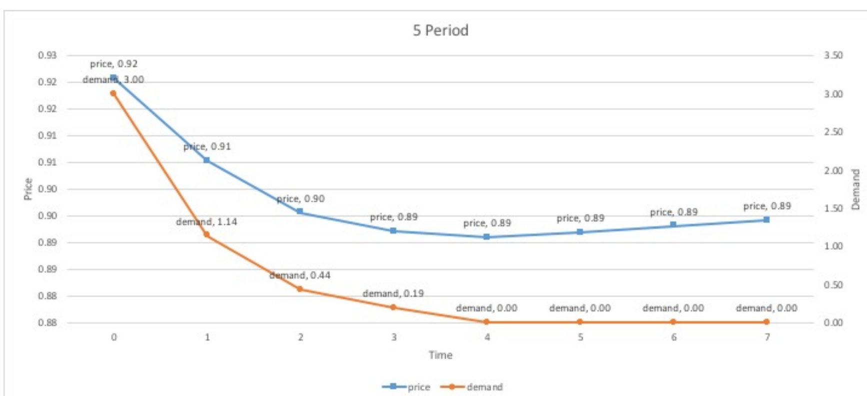
A.3 Optimal Liquidation Strategies



(a) 3 Periods Trading Strategy, Cost = 0.0625

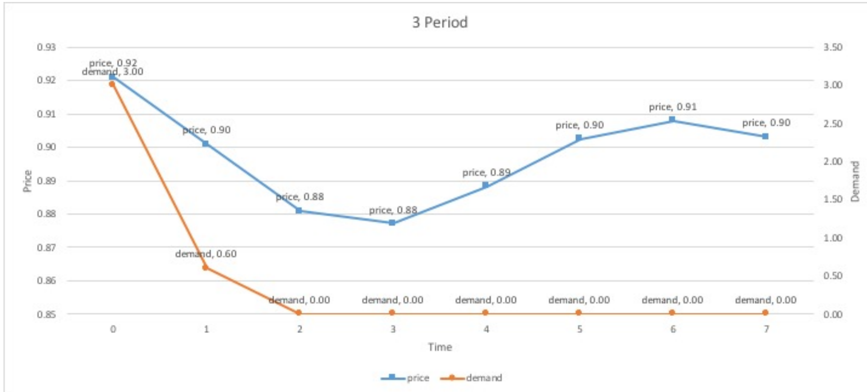


(b) 4 Periods Trading Strategy, Cost = 0.0601

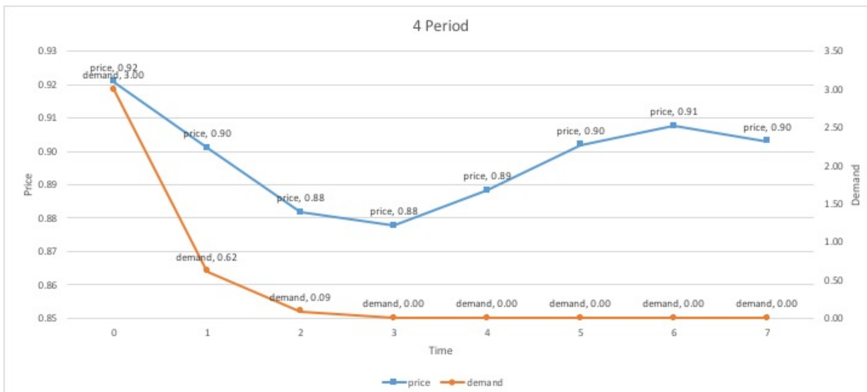


(c) 5 Periods Trading Strategy, Cost = 0.0594

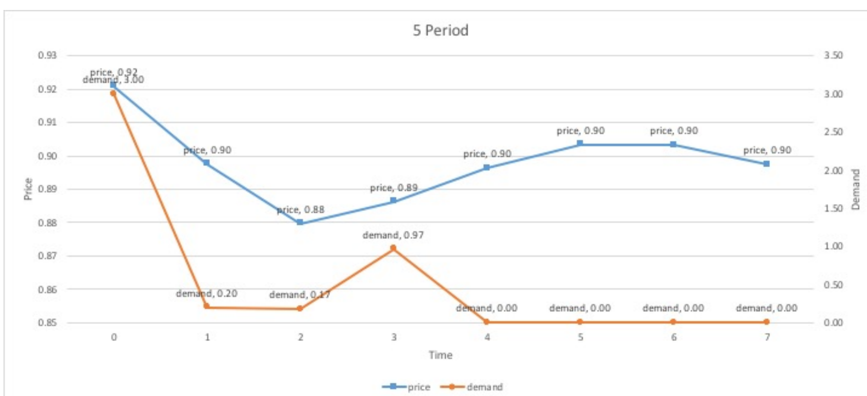
Figure A.1: Optimal Liquidation Strategy with no fundamental shock, no initial sentiment, and θ equals to 0.75. The liquidation cost decreases with longer trading horizons. Equilibrium prices first drop and then gradually recover.



(a) 3 Periods Trading Strategy, Cost = 0.0720

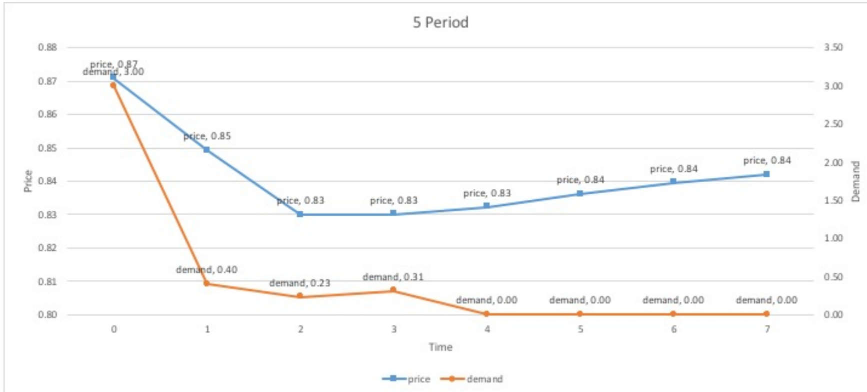


(b) 4 Periods Trading Strategy, Cost = 0.0718

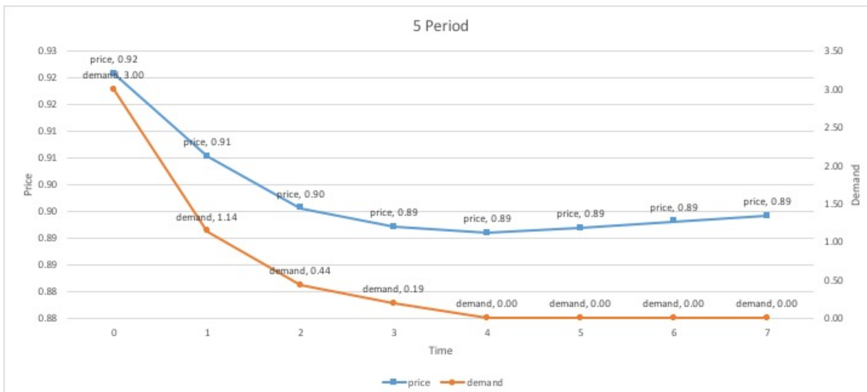


(c) 5 Periods Trading Strategy, Cost = 0.0627

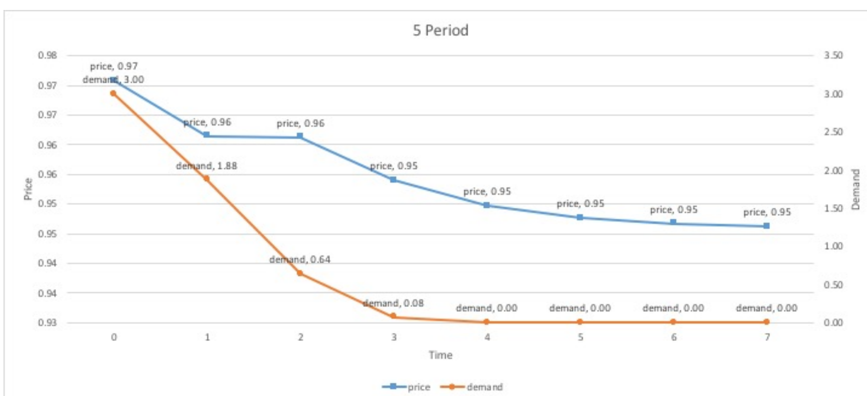
Figure A.2: Optimal Liquidation Strategy with no fundamental shock, no initial sentiment, and θ equals to 0.25. The liquidation cost decreases with longer trading horizons. Equilibrium prices first drop and then rapidly recover.



(a) Negative Initial Shock, Cost = 0.0720

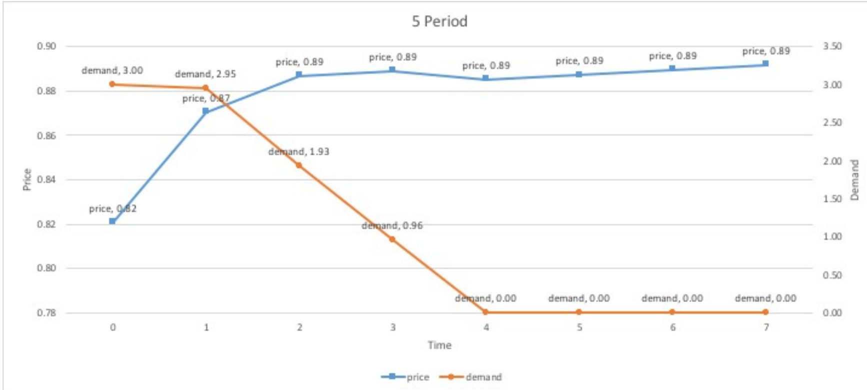


(b) Zero Initial Shock, Cost = 0.0594

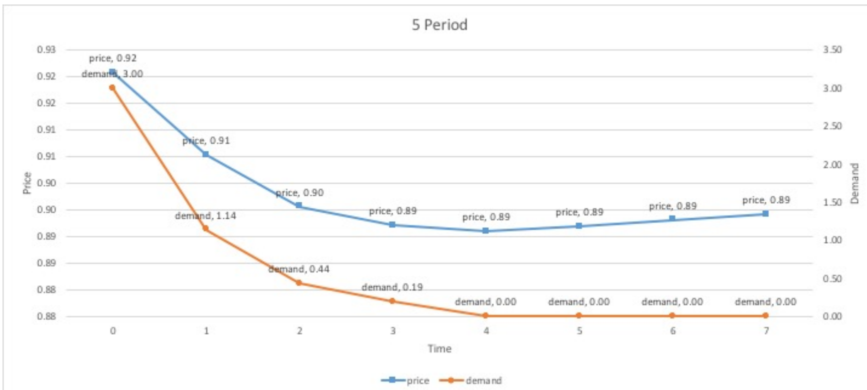


(c) Positive Initial Shock, Cost = 0.0333

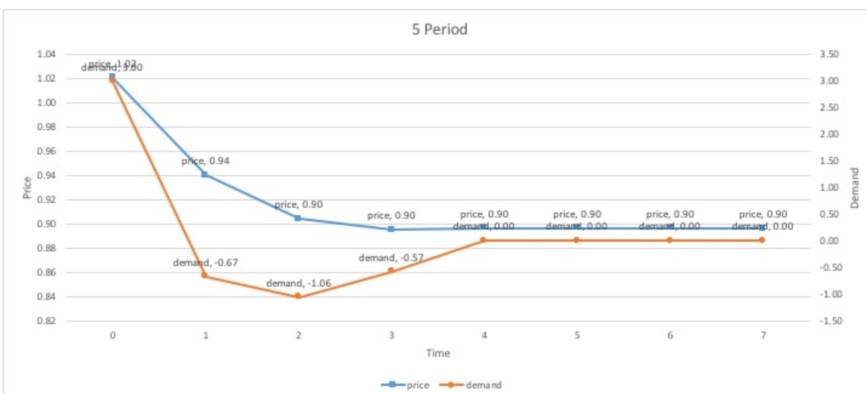
Figure A.3: 5 Periods Optimal Liquidation Strategy with no initial sentiment, and θ equals to 0.75. Initial fundamental shock increases from -0.05 to $+0.05$. The liquidation cost decreases correspondingly. Equilibrium prices first drop and then recover.



(a) Negative Initial Sentiment, Profit = 0.1979



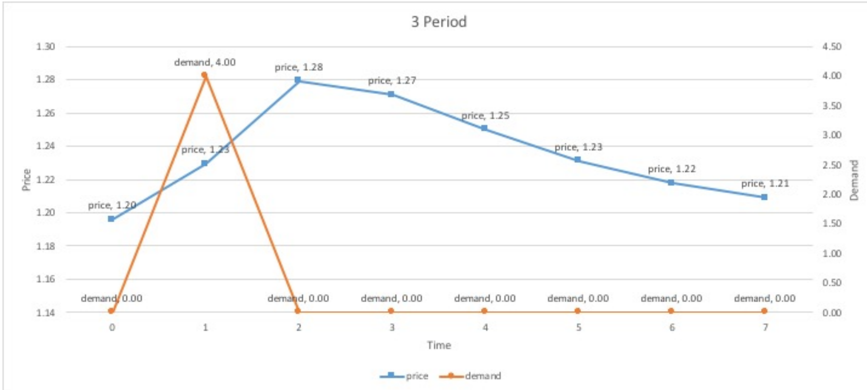
(b) Zero Initial Sentiment, Cost = 0.0594



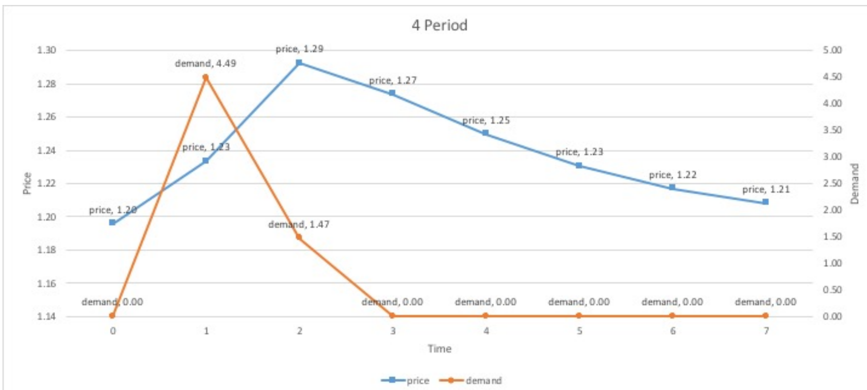
(c) Positive Initial Sentiment, Cost = 0.2087

Figure A.4: 5 Periods Optimal Liquidation Strategy with no fundamental shock, and θ equals to 0.75. Initial sentiment switches from negative to positive. The liquidation cost increases correspondingly.

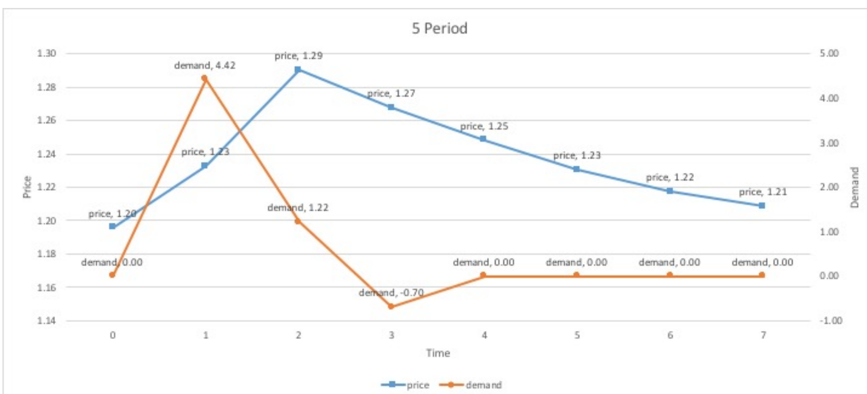
A.4 Market Manipulation Strategies



(a) 3 Periods Trading Strategy, Profit = 0.2000

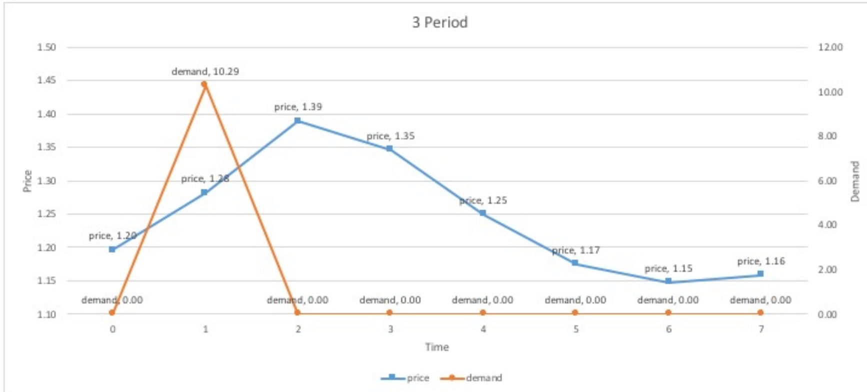


(b) 4 Periods Trading Strategy, Profit = 0.2384

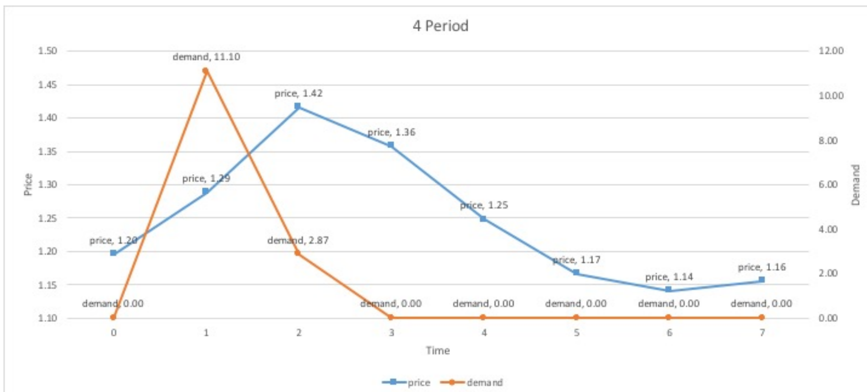


(c) 5 Periods Trading Strategy, Profit = 0.2402

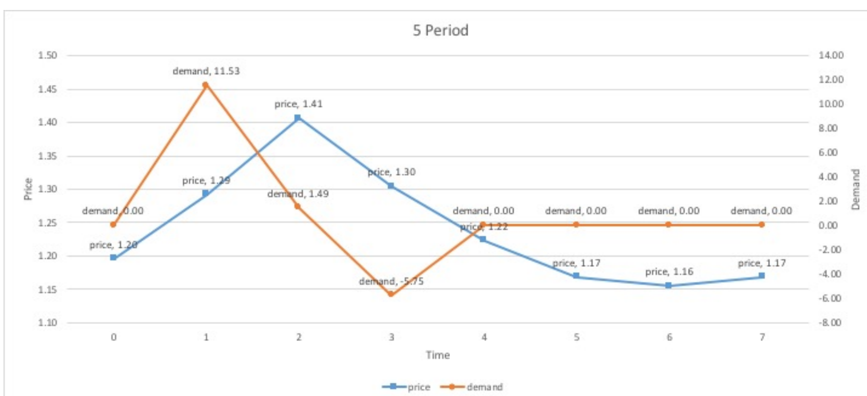
Figure A.5: Market Manipulation Strategy with positive initial fundamental shock, no initial sentiment, and θ equals to 0.75. An asset price bubble is created. The profit increases with longer trading horizons.



(a) 3 Periods Trading Strategy, Profit = 1.1020

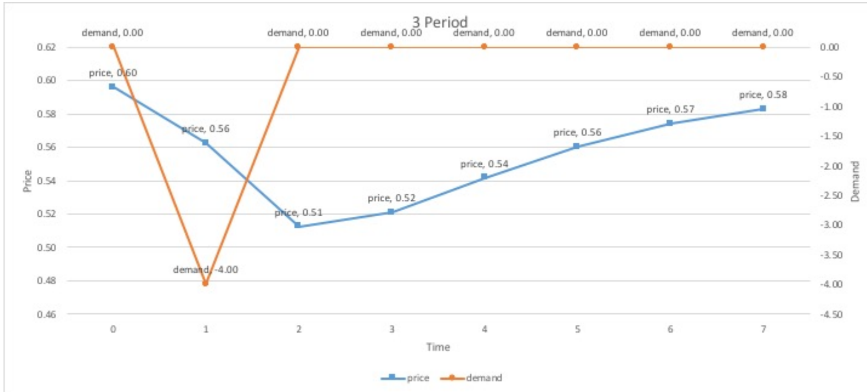


(b) 4 Periods Trading Strategy, Profit = 1.2503

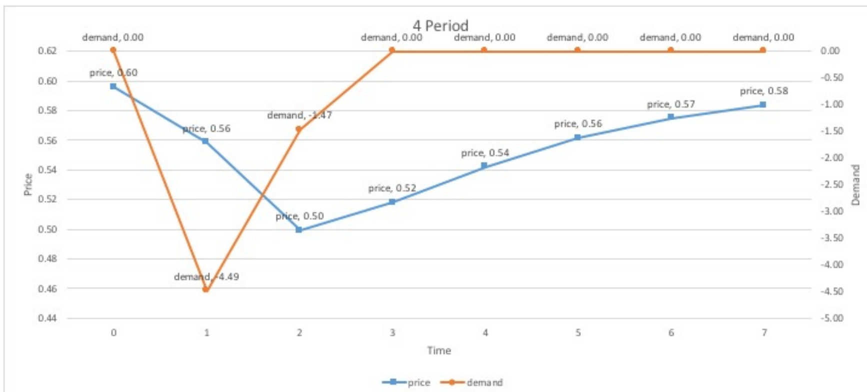


(c) 5 Periods Trading Strategy, Profit = 1.6340

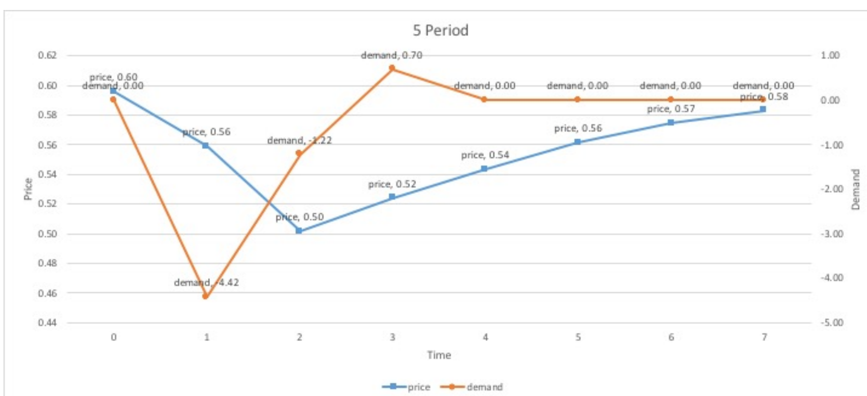
Figure A.6: Market Manipulation Strategy with positive initial fundamental shock, no initial sentiment, and θ equals to 0.5. A larger asset price bubble is created. The profit increases with longer trading horizons.



(a) 3 Periods Trading Strategy, Profit = 1.1020

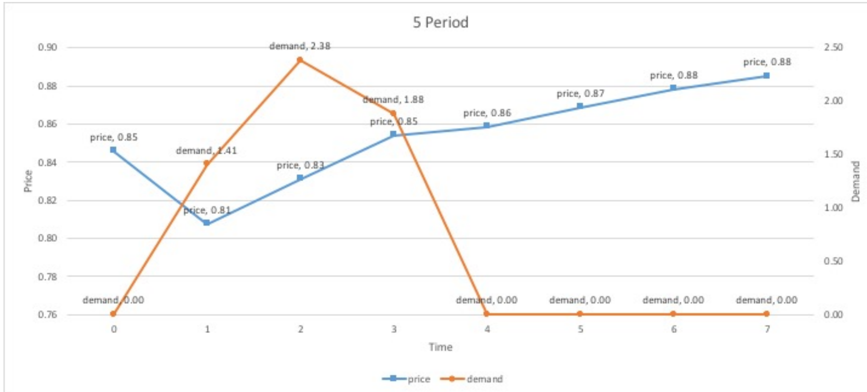


(b) 4 Periods Trading Strategy, Profit = 1.2503

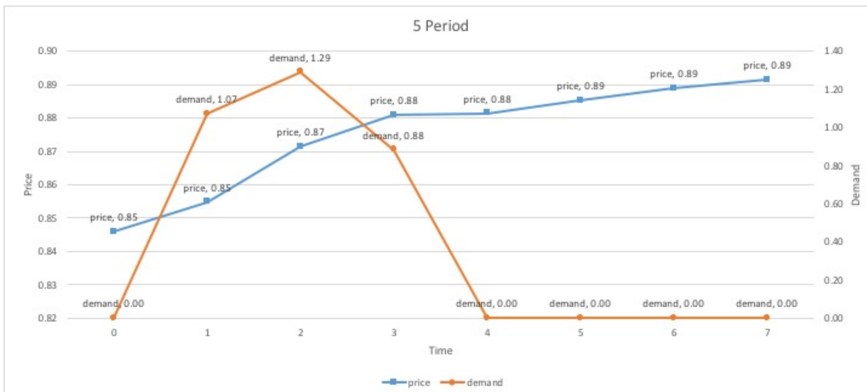


(c) 5 Periods Trading Strategy, Profit = 1.6340

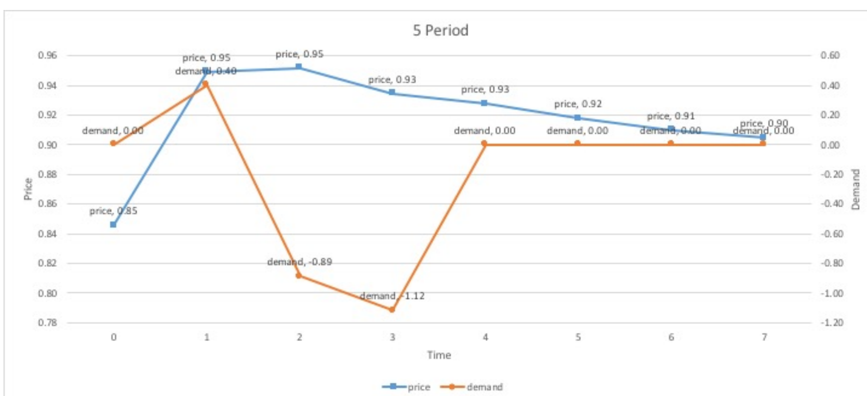
Figure A.7: Market Manipulation Strategy with negative initial fundamental shock, no initial sentiment, and θ equals to 0.75. A negative asset price bubble is created. Bear raid strategy is stimulated. The profit increases with longer trading horizons.



(a) Decreasing initial sentiment, Profit = 0.0959



(b) Flat initial sentiment, Profit = 0.0306



(c) Increasing initial sentiment, Profit = 0.0240

Figure A.8: Market Manipulation Strategy with no initial fundamental shock and θ equals to 0.75. Sentiment at time 0 is negative. In panel (a) sentiment decreases at time 1 and deviate further from zero; in panel (b) sentiment is flat; in panel (c) sentiment turns positive.

APPENDIX B

APPENDIX OF CHAPTER 3

B.1 Proofs of Benchmark Model

B.1.1 Proof of Proposition 4

The rational agent maximizes his expected utility at time 0

$$\mathbb{E}_0^r\left[-\int_0^\infty \frac{e^{-\delta t-\gamma C_t^r}}{\gamma}dt\right]$$

subject to the budget constraint

$$dW_t^r = (rW_t^r - C_t^r)dt + N_t^r[(D_t - rP_t^r)dt + dP_t^r]$$

Assuming the derived value function is

$$J_t^r(W_t^r;t) \equiv \max_{\{C_s^r,N_s^r\}_{s\geq t}} \mathbb{E}_t^r\left[-\int_t^\infty \frac{e^{-\delta s-\gamma C_s^r}}{\gamma}ds\right]$$

Using Ito's lemma and HJB equation, we are able to obtain the following PDE:

$$0 = -\frac{e^{-\delta t-\gamma C_t^r}}{\gamma} - \delta J^r + J_W^r dW^r + \frac{1}{2}J_{WW}^r dW^r dW^r \tag{B.1}$$

We conjecture, and later verify, that the value function and prices are given by

$$\begin{aligned} P_t^r &= p_0^r + \frac{D_t}{r} \\ J_t^r &= -e^{-\delta t - r\gamma W_t^r + c^r} \end{aligned}$$

So the PDE (B.1) can be transformed into

$$0 = r - \delta - r[c^r + log(r\gamma) + \gamma N_t^r(-rp_0^r + \frac{g_D}{r})] + \frac{1}{2}(r\gamma)^2(\frac{\sigma_D}{r})^2(N_t^r)^2 \tag{B.2}$$

First order condition with respect to N_t^r gives

$$0 = -r\gamma(-rp_0^r + \frac{g_D}{r}) + (r\gamma)^2(\frac{\sigma_D}{r})^2 N_t^r$$

Plug in the optimal portfolio strategy $N_t^r = Q$, we are able to solve for p_0^r such that

$$p_0^r = -r(\frac{\sigma_D}{r})^2 Q + \frac{g_D}{r^2} \tag{B.3}$$

Lastly, plug (B.3) into (B.2) to solve for the constant c^r, the value function is then

$$J_t^r = -e^{-\delta t - r\gamma W_t^r + \frac{r-\delta}{r} - \frac{\gamma^2\sigma_D^2 Q^2}{2r} - log(r\gamma)} \tag{B.4}$$

This completes the proof of Proposition 1. ■

B.1.2 Proof of Corollary 4

The differential form of the price function gives:

$$dP_t^r = \frac{g_D}{r}dt + \frac{\sigma_D}{r}dZ_t^D$$

Thus $\sigma_P^r = \frac{\sigma_D}{r}$.

The risk premium is calculated as

$$\frac{\mathbb{E}[(D_t - rP_t^r)dt + dP_t^r]}{dt} = \frac{\gamma Q\sigma_D^2}{r}$$

This completes the proof. ■

B.2 Proofs of MAX-CAPM Model

B.2.1 Extrapolators' Optimization Problem

Let's first set up the optimization problem. Each extrapolator maximizes

$$\mathbb{E}_0^e\left[-\int_0^\infty \frac{e^{-\delta t-\gamma C_t^e}}{\gamma}dt\right]$$

subject to the budget constraint and state processes

$$\begin{aligned}
dW_t^e &= (rW_t^e - C_t^e)dt + N_t^e[(D_t - rP_t)dt + dP_t^e] \\
&= (rW_t^e - C_t^e)dt + N_t^e[(e_0^e + e_S^e S_t + e_{N^a}^e N_t^a)dt + \vec{\sigma}_P d\vec{Z}_t^e] \\
dS_t &= \beta(-S_t)dt + \beta(\lambda_0 + \lambda_1 S_t)dt + \beta\vec{\sigma}_P d\vec{Z}_t^e \\
&= \beta[\lambda_0 + (\lambda_1 - 1)S_t]dt + \beta\vec{\sigma}_P d\vec{Z}_t^e \\
dN_t^a &= \alpha(S_t - N_t^a)dt + \sigma_a d\hat{Z}_t^a
\end{aligned}$$

where $e_0^e = \lambda_0 - rp_0$, $e_S^e = \lambda_1 - rp_S$, $e_{N^a}^e = -rp_a$.

In order to find the diffusion parameters $\vec{\sigma}_P$, we first differentiate the conjectured price function

$$dP_t = p_S dS_t + \frac{1}{r}dD_t + p_a dN_t^a$$

We then plug in the processes for dividend, sentiment, and arbitrageurs' true demand. The diffusion term of the price function is then

$$\vec{\sigma}_P d\vec{Z}_t = \frac{1}{1-\beta p_S}\left(\frac{\sigma_D}{r}dZ_t^D + p_a\sigma_a dZ_t^a\right)$$

and

$$\vec{\sigma}_P d\vec{Z}_t^e = \frac{1}{1-\beta p_S}\left(\frac{\sigma_D}{r} d\hat{Z}_t^D + p_a \sigma_a d\hat{Z}_t^a\right)$$

Assuming the derived value function for the extrapolators is

$$J_t^e(W_t^e; S_t, N_t^a; t) \equiv \max_{\{C_s^e, N_s^e\}_{s\geq t}} \mathbb{E}_t^e\left[-\int_t^\infty \frac{e^{-\delta s - \gamma C_s^e}}{\gamma} ds\right]$$

Using Ito's lemma and HJB equation, we are able to obtain the following PDE:

$$\begin{aligned}
0 = & -\frac{e^{-\delta t - \gamma C_t^e}}{\gamma} - \delta J^e + J_W^e dW^e + J_S^e dS + J_{N^a}^e dN^a \\
& + \frac{1}{2} J_{WW}^e dW^e dW^e + \frac{1}{2} J_{SS}^e dS\, dS + \frac{1}{2} J_{N^a N^a}^e dN^a dN^a \\
& + J_{WS}^e dW^e dS + J_{WN^a}^e dW^e dN^a + J_{SN^a}^e dS\, dN^a
\end{aligned} \tag{B.5}$$

Now we conjecture the value function J_t^e has the form:

$$J_t^e = -e^{-\delta t - r\gamma W^e + a^e S^2 + b^e (N^a)^2 + c^e S N^a + d^e S + e^e N^a + f^e}$$

The we can calculate the partial differentials:

$$\begin{aligned}
J_W^e &= -r\gamma J^e \\
J_S^e &= (2a^e S + c^e N^a + d^e) J^e \\
J_{N^a}^e &= (2b^e N^a + c^e S + e^e) J^e \\
J_{WW}^e &= (r\gamma)^2 J^e \\
J_{SS}^e &= [2a^e + (2a^e S + c^e N^a + d^e)^2] J^e \\
J_{N^a N^a}^e &= [2b^e + (2b^e N^a + c^e S + e^e)^2] J^e \\
J_{WS}^e &= -r\gamma(2a^e S + c^e N^a + d^e) J^e \\
J_{WN^a}^e &= -r\gamma(2b^e N^a + c^e S + e^e) J^e \\
J_{SN^a}^e &= [c^e + (2a^e S + c^e N^a + d^e)(2b^e N^a + c^e S + e^e)] J^e
\end{aligned}$$

Thus equation B.5 can be transformed into

$$
\begin{aligned}
0 = {} & r - \delta \\
& - r\Big[a^e S_t^2 + b^e (N_t^a)^2 + c^e S_t N_t^a + d^e S_t + e^e N_t^a + f^e + \log(r\gamma) + \gamma N_t^e (e_0^e + e_S^e S_t + e_{N^a}^e N_t^a)\Big] \\
& + \beta(2a^e S_t + c^e N_t^a + d^e)(\lambda_0 + (\lambda_1 - 1)S_t) + \alpha(2b^e N_t^a + c^e S_t + e^e)(S_t - N_t^a) \\
& + \frac{1}{2}(r\gamma)^2 \vec{\sigma_P}^2 (N_t^e)^2 + \frac{1}{2}[2a^e + (2a^e S_t + c^e N_t^a + d^e)^2]\beta^2 \vec{\sigma_P}^2 \\
& + \frac{1}{2}\sigma_a^2 [2b^e + (2b^e N_t^a + c^e S_t + e^e)^2] \\
& - r\gamma\beta(2a^e S_t + c^e N_t^a + d^e)\vec{\sigma_P}^2 N_t^e - r\gamma(2b^e N_t^a + c^e S_t + e^e)\frac{p_a \sigma_a^2}{1 - p_S \beta} N_t^e \\
& + [c^e + (2a^e S_t + c^e N_t^a + d^e)(2b^e N_t^a + c^e S_t + e^e)]\beta \frac{p_a \sigma_a^2}{1 - p_S \beta}
\end{aligned}
\tag{B.6}
$$

Take first order condition with respect to N_t^e, we obtain the optimal portfolio strategy

$$
\begin{aligned}
0 = {} & - r\gamma(e_0^e + e_S^e S_t + e_{N^a}^e N_t^a) + (r\gamma)^2 \vec{\sigma_P}^2 N_t^e - r\gamma\beta(2a^e S_t + c^e N_t^a + d^e)\vec{\sigma_P}^2 \\
& - r\gamma(2b^e N_t^a + c^e S_t + e^e)\frac{p_a \sigma_a^2}{1 - p_S \beta}
\end{aligned}
$$

Rearrange the above equation, we can represent N_t^e in terms of a linear combination of our state variables S_t and N_t^a as

$$
N_t^e = f_0^e + f_S^e S_t + f_{N^a}^e N_t^a \tag{B.7}
$$

where

$$
\begin{aligned}
f_0^e &= \frac{e_0^e + \beta d^e \vec{\sigma_P}^2 + e^e \frac{p_a \sigma_a^2}{1 - p_S \beta}}{r\gamma \vec{\sigma_P}^2} \\
f_S^e &= \frac{e_S^e + 2a^e \beta \vec{\sigma_P}^2 + c^e \frac{p_a \sigma_a^2}{1 - p_S \beta}}{r\gamma \vec{\sigma_P}^2} \\
f_{N^a}^e &= \frac{e_{N^a}^e + c^e \beta \vec{\sigma_P}^2 + 2b^e \frac{p_a \sigma_a^2}{1 - p_S \beta}}{r\gamma \vec{\sigma_P}^2}
\end{aligned}
$$

Plug the optimal portfolio strategy back into the equation B.6, we obtain the following quadratic equation in S and N^a

$$
\begin{aligned}
0 = {} & r - \delta \\
& - r[a^e S_t^2 + b^e (N_t^a)^2 + c^e S_t N_t^a + d^e S_t + e^e N_t^a + f^e + \log(r\gamma)] \\
& - r\gamma (f_0^e + f_S^e S_t + f_{N^a}^e N_t^a)(e_0^e + e_S^e S_t + e_{N^a}^e N_t^a) \\
& + \beta(2a^e S_t + c^e N_t^a + d^e)(\lambda_0 + (\lambda_1 - 1)S_t) + \alpha(2b^e N_t^a + c^e S_t + e^e)(S_t - N_t^a) \\
& + \frac{1}{2}(r\gamma)^2 \vec{\sigma_P}^2 (f_0^e + f_S^e S_t + f_{N^a}^e N_t^a)^2 + \frac{1}{2}[2a^e + (2a^e S_t + c^e N_t^a + d^e)^2]\beta^2 \vec{\sigma_P}^2 \\
& + \frac{1}{2}\sigma_a^2 [2b^e + (2b^e N_t^a + c^e S_t + e^e)^2] \\
& - r\gamma\beta \vec{\sigma_P}^2 (2a^e S_t + c^e N_t^a + d^e)(f_0^e + f_S^e S_t + f_{N^a}^e N_t^a) \\
& - r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta}(2b^e N_t^a + c^e S_t + e^e)(f_0^e + f_S^e S_t + f_{N^a}^e N_t^a) \\
& + \beta \frac{p_a \sigma_a^2}{1 - p_S \beta}[c^e + (2a^e S_t + c^e N_t^a + d^e)(2b^e N_t^a + c^e S_t + e^e)]
\end{aligned}
\tag{B.8}
$$

The above equation is valid for any value of S_t and N_t^a, which implies the coefficients of variables should be zero. That is

$$
\begin{aligned}
0 &= -ra^e - r\gamma f_S^e e_S^e + 2a^e\beta(\lambda_1 - 1) + c^e\alpha + \frac{1}{2}(r\gamma)^2 f_S^{e2}\vec{\sigma_P}^2 + 2a^{e2}\beta^2\vec{\sigma_P}^2 + \frac{1}{2}c^{e2}\sigma_a^2 - 2a^e f_S^e r\gamma\beta\vec{\sigma_P}^2 \\
&\quad - c^e f_S^e r\gamma\frac{p_a\sigma_a^2}{1 - p_S\beta} + 2a^e c^e\beta\frac{p_a\sigma_a^2}{1 - p_S\beta} \\
0 &= -rb^e - r\gamma f_{N^a}^e e_{N^a}^e - 2b^e\alpha + \frac{1}{2}(r\gamma)^2 f_{N^a}^{e\;2}\vec{\sigma_P}^2 + \frac{1}{2}c^{e2}\beta^2\vec{\sigma_P}^2 + 2b^{e2}\sigma_a^2 - c^e f_{N^a}^e r\gamma\beta\vec{\sigma_P}^2 \\
&\quad - 2b^e f_{N^a}^e r\gamma\frac{p_a\sigma_a^2}{1 - p_S\beta} + 2b^e c^e\beta\frac{p_a\sigma_a^2}{1 - p_S\beta} \\
0 &= -rc^e - r\gamma(f_S^e e_{N^a}^e + f_{N^a}^e e_S^e) + \beta c^e(\lambda_1 - 1) + \alpha(2b^e - c^e) + (r\gamma)^2 f_S^e f_{N^a}^e\vec{\sigma_P}^2 + 2a^e c^e\beta^2\vec{\sigma_P}^2 \\
&\quad + 2b^e c^e\sigma_a^2 - r\gamma\beta\vec{\sigma_P}^2(2a^e f_{N^a}^e + c^e f_S^e) - r\gamma\frac{p_a\sigma_a^2}{1 - p_S\beta}(2b^e f_S^e + c^e f_{N^a}^e) + \beta\frac{p_a\sigma_a^2}{1 - p_S\beta}(4a^e b^e + c^{e2}) \\
0 &= -rd^e - r\gamma(f_0^e e_S^e + f_S^e e_0^e) + \beta(2a^e\lambda_0 + d^e(\lambda_1 - 1)) + \alpha e^e + (r\gamma)^2\vec{\sigma_P}^2 f_0^e f_S^e + 2a^e d^e\beta^2\vec{\sigma_P}^2 + c^e e^e\sigma_a^2 \\
&\quad - r\gamma\beta\vec{\sigma_P}^2(2a^e f_0^e + d^e f_S^e) - r\gamma\frac{p_a\sigma_a^2}{1 - p_S\beta}(c^e f_0^e + e^e f_S^e) + \beta\frac{p_a\sigma_a^2}{1 - p_S\beta}(2a^e e^e + c^e d^e) \\
0 &= -re^e - r\gamma(f_0^e e_{N^a}^e + f_{N^a}^e e_0^e) + \beta c^e\lambda_0 - \alpha e^e + (r\gamma)^2\vec{\sigma_P}^2 f_0^e f_{N^a}^e + c^e d^e\beta^2\vec{\sigma_P}^2 + 2b^e e^e\sigma_a^2 \\
&\quad - r\gamma\beta\vec{\sigma_P}^2(c^e f_0^e + d^e f_{N^a}^e) - r\gamma\frac{p_a\sigma_a^2}{1 - p_S\beta}(2b^e f_0^e + e^e f_{N^a}^e) + \beta\frac{p_a\sigma_a^2}{1 - p_S\beta}(c^e e^e + 2b^e d^e) \\
0 &= r - \delta - rf^e - r\log(r\gamma) - r\gamma f_0^e e_0^e + \beta d^e\lambda_0 + \frac{1}{2}(r\gamma)^2 f_0^{e2}\vec{\sigma_P}^2 + \frac{1}{2}\beta^2\vec{\sigma_P}^2(2a^e + d^{e2}) \\
&\quad + \frac{1}{2}\sigma_a^2(2b^e + e^{e2}) - r\gamma\beta d^e f_0^e\vec{\sigma_P}^2 - r\gamma\frac{p_a\sigma_a^2}{1 - p_S\beta}e^e f_0^e + \beta\frac{p_a\sigma_a^2}{1 - p_S\beta}(c^e + d^e e^e)
\end{aligned}
\tag{B.9}
$$

Solving this quadratic equations system, we are able to determine the coefficients, so as to pin down the optimal consumption and portfolio policies for extrapolators in terms of state variables S_t and N_t^a.

B.2.2 Monopolistic Traders' Optimization Problem CASE (a)

In this case the monopolistic trader are more like a "smart trader" or "knowledgeable trader" in the sense that he has correct beliefs. The monopolistic traders have infinite horizon at $[0, \infty)$. He tries to maximize the expected utility on the intial time 0

$$\mathbb{E}_0^m\left[-\int_0^\infty \frac{e^{-\delta t-\gamma C_t^m}}{\gamma}dt\right]$$

subject to the budget constraint and state processes

$$\begin{aligned} dW_t^m &= (rW_t^m - C_t^m)dt + N_t^m[(D_t - rP_t)dt + dP_t] \\ dS_t &= -\beta S_t dt + \beta dP_t \\ dN_t^a &= \alpha[(P_t^r - P_t) - N_t^a]dt + \sigma_a dZ_t^a \end{aligned}$$

where P_t is the true price process of risky asset and N_t^a is the arbitrageurs' true demand.

In order to characterize investment opportunity in this economy, again we consider the instantaneous excess return to one share of stock: $dQ_t^m = (D_t - rP_t)dt + dP_t$, which can also be thought as the return on a zero-wealth portfolio long one share of stock fully financed by borrowing at the risk-free rate.

$$
\begin{aligned}
dP_t &= p_S dS_t + \frac{1}{r}D_t + p_a dN_t^a \\
&= -p_S\beta S_t dt + p_S\beta dP_t + \frac{1}{r}g_D dt + \frac{1}{r}\sigma_D dZ_t^D + p_a\alpha(P_t^r - P_t - N_t^a)dt + p_a\sigma_a dZ_t^a \\
&= \frac{1}{1-p_S\beta}\left[\frac{g_D}{r} + p_a\alpha(p_0^r - p_0) - (p_S\beta + p_a p_S\alpha)S_t - p_a(p_a+1)\alpha N_t^a\right]dt \\
&\quad + \frac{1}{1-p_S\beta}\left(\frac{\sigma_D}{r}dZ_t^D + p_a\sigma_a dZ_t^a\right) \\
dQ_t^m &= (D_t - rP_t)dt + dP_t \\
&= [(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a)]dt + \vec{\sigma}_P d\vec{Z}_t
\end{aligned}
$$

Where $\vec{\sigma}_P d\vec{Z}_t$ is specified before and e_0^m, e_S^m, and $e_{N^a}^m$ has the following form:

$$
\begin{aligned}
e_0^m &= -rp_0 + \frac{g_D + \alpha r p_a(p_0^r - p_0)}{r(1-p_S\beta)} \\
e_S^m &= -rp_S - \frac{p_S\beta + p_a p_S\alpha}{1-p_S\beta} \\
e_{N^a}^m &= -rp_a - \frac{p_a(p_a+1)\alpha}{1-p_S\beta}
\end{aligned}
$$

The monopolistic traders' optimization problem now becomes

$$
\mathbb{E}_0^m\left[-\int_0^\infty \frac{e^{-\delta t - \gamma C_t^m}}{\gamma}dt\right]
$$

subject to the budget constraint

$$
\begin{aligned}
dW_t^m &= (rW_t^m - C_t^m)dt + N_t^m dQ_t^m \\
&= (rW_t^m - C_t^m)dt + N_t^m(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a)dt + N_t^m\vec{\sigma}_P d\vec{Z}_t \\
dS_t &= \frac{\beta}{1-p_S\beta}\left[\frac{g_D}{r} + p_a\alpha(p_0^r - p_0) - (1 + p_a p_S\alpha)S_t - p_a(p_a+1)\alpha N_t^a\right]dt + \beta\vec{\sigma}_P d\vec{Z}_t \\
&= (\phi_0^m + \phi_S^m S_t + \phi_{N^a}^m N_t^a)dt + \beta\vec{\sigma}_P d\vec{Z}_t \\
dN_t^a &= \alpha[(p_0^r - p_0) - p_S S_t - (p_a+1)N_t^a]dt + \sigma_a dZ_t^a
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_0^m &= \frac{\beta}{1-p_S\beta}\frac{g_D+\alpha r p_a(p_0^r-p_0)}{r} \\
\phi_S^m &= -\frac{\beta}{1-p_S\beta}(1+p_a p_S\alpha) \\
\phi_{N^a}^m &= -\frac{\beta}{1-p_S\beta}p_a(p_a+1)\alpha
\end{aligned}
$$

Assuming the derived value function for the extrapolators is

$$
J_t^m(W_t^m;S_t,N_t^a;t) \equiv \max_{\{C_s^m,N_s^m\}_{s\geq t}} \mathbb{E}_t^m\left[-\int_t^\infty \frac{e^{-\delta s-\gamma C_s^m}}{\gamma}ds\right]
$$

Using Ito's lemma and HJB equation, we are able to obtain the following PDE:

$$
\begin{aligned}
0 = & -\frac{e^{-\delta t-\gamma C_t^m}}{\gamma} - \delta J^m + J_W^m dW^m + J_S^m dS + J_{N^a}^m dN^a \\
& + \frac{1}{2}J_{WW}^m dW^m dW^m + \frac{1}{2}J_{SS}^m dS\, dS + \frac{1}{2}J_{N^aN^a}^m dN^a dN^a \\
& + J_{WS}^m dW^m dS + J_{WN^a}^m dW^m dN^a + J_{SN^a}^m dS\, dN^a
\end{aligned} \tag{B.10}
$$

We conjecture, and later verify, that the value function J_t^m has the following form:

$$
J_t^m = -e^{-\delta t - r\gamma W^m + a^m S^2 + b^m (N^a)^2 + c^m S N^a + d^m S + e^m N^a + f^m}
$$

The we can calculate the partial differentials:

$$
\begin{aligned}
J_W^m &= -r\gamma J^m \\
J_S^m &= (2a^m S + c^m N^a + d^m) J^m \\
J_{N^a}^m &= (2b^m N^a + c^m S + e^m) J^m \\
J_{WW}^m &= (r\gamma)^2 J^m \\
J_{SS}^m &= [2a^m + (2a^m S + c^m N^a + d^m)^2] J^m \\
J_{N^a N^a}^m &= [2b^m + (2b^m N^a + c^m S + e^m)^2] J^m \\
J_{WS}^m &= -r\gamma(2a^m S + c^m N^a + d^m) J^m \\
J_{WN^a}^m &= -r\gamma(2b^m N^a + c^m S + e^m) J^m \\
J_{SN^a}^m &= [c^m + (2a^m S + c^m N^a + d^m)(2b^m N^a + c^m S + e^m)] J^m
\end{aligned}
$$

Thus equation B.10 can be transformed into

$$
\begin{aligned}
0 = r - \delta& \\
&- r\left[a^m S_t^2 + b^m (N_t^a)^2 + c^m S_t N_t^a + d^m S_t + e^m N_t^a + f^m + \log(r\gamma) + \gamma N_t^m (e_0^m + e_S^m S_t + e_{N^a}^m N_t^a)\right] \\
&+ (2a^m S_t + c^m N_t^a + d^m)(\phi_0^m + \phi_S^m S_t + \phi_{N^a}^m N_t^a) \\
&+ \alpha(2b^m N_t^a + c^m S_t + e^m)[(p_0^r - p_0) - p_S S_t - (p_a + 1) N_t^a] \\
&+ \frac{1}{2}(r\gamma)^2 \vec{\sigma_P}^2 (N_t^m)^2 + \frac{1}{2}[2a^m + (2a^m S_t + c^m N_t^a + d^m)^2]\beta^2 \vec{\sigma_P}^2 \\
&+ \frac{1}{2}\sigma_a^2 [2b^m + (2b^m N_t^a + c^m S_t + e^m)^2] \\
&- r\gamma\beta(2a^m S_t + c^m N_t^a + d^m)\vec{\sigma_P}^2 N_t^m - r\gamma(2b^m N_t^a + c^m S_t + e^m)\frac{p_a \sigma_a^2}{1 - p_S \beta} N_t^m \\
&+ [c^m + (2a^m S_t + c^m N_t^a + d^m)(2b^m N_t^a + c^m S_t + e^m)]\beta \frac{p_a \sigma_a^2}{1 - p_S \beta}
\end{aligned}
\tag{B.11}
$$

Take first order condition with respect to N_t^e, we obtain the optimal portfolio strategy

$$0 = -r\gamma(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a) + (r\gamma)^2 \vec{\sigma_P}^2 N_t^m - r\gamma\beta(2a^m S_t + c^m N_t^a + d^m)\vec{\sigma_P}^2 - r\gamma(2b^m N_t^a + c^m S_t + e^m)\frac{p_a\sigma_a^2}{1 - p_S\beta}$$

Rearrange the above equation, we can represent N_t^m in terms of a linear combination of our state variables S_t and N_t^a as

$$N_t^m = f_0^m + f_S^m S_t + f_{N^a}^m N_t^a \tag{B.12}$$

where

$$\begin{aligned} f_0^m &= \frac{e_0^m + \beta d^m \vec{\sigma_P}^2 + e^m \frac{p_a\sigma_a^2}{1-p_S\beta}}{r\gamma\vec{\sigma_P}^2} \\ f_S^m &= \frac{e_S^m + 2a^m\beta\vec{\sigma_P}^2 + c^m \frac{p_a\sigma_a^2}{1-p_S\beta}}{r\gamma\vec{\sigma_P}^2} \\ f_{N^a}^m &= \frac{e_{N^a}^m + c^m\beta\vec{\sigma_P}^2 + 2b^m \frac{p_a\sigma_a^2}{1-p_S\beta}}{r\gamma\vec{\sigma_P}^2} \end{aligned}$$

Plug the optimal portfolio strategy back into equation B.11, we obtain the following quadratic equation in S and N^a

$$
\begin{aligned}
0 = {} & r - \delta \\
& - r[a^m S_t^2 + b^m (N_t^a)^2 + c^m S_t N_t^a + d^m S_t + e^m N_t^a + f^m + \log(r\gamma)] \\
& - r\gamma(f_0^m + f_S^m S_t + f_{N^a}^m N_t^a)(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a) \\
& + (2a^m S_t + c^m N_t^a + d^m)(\phi_0^m + \phi_S^m S_t + \phi_{N^a}^m N_t^a) \\
& + \alpha(2b^m N_t^a + c^m S_t + e^m)[(p_0^r - p_0) - p_S S_t - (p_a + 1)N_t^a] \\
& + \frac{1}{2}(r\gamma)^2 \vec{\sigma_P}^2 (f_0^m + f_S^m S_t + f_{N^a}^m N_t^a)^2 + \frac{1}{2}[2a^m + (2a^m S_t + c^m N_t^a + d^m)^2]\beta^2 \vec{\sigma_P}^2 \\
& + \frac{1}{2}\sigma_a^2 [2b^m + (2b^m N_t^a + c^m S_t + e^m)^2] \\
& - r\gamma\beta\vec{\sigma_P}^2 (2a^m S_t + c^m N_t^a + d^m)(f_0^m + f_S^m S_t + f_{N^a}^m N_t^a) \\
& - r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta}(2b^m N_t^a + c^m S_t + e^m)(f_0^m + f_S^m S_t + f_{N^a}^m N_t^a) \\
& + \beta \frac{p_a \sigma_a^2}{1 - p_S \beta}[c^m + (2a^m S_t + c^m N_t^a + d^m)(2b^m N_t^a + c^m S_t + e^m)]
\end{aligned}
\tag{B.13}
$$

The above equation is valid for any value of S_t and N_t^a, which implies the coefficients of variables should be zero. That is

$$
\begin{aligned}
0 = {} & -ra^m - r\gamma f_S^m e_S^m + 2a^m \phi_S^m - c^m p_S \alpha + \frac{1}{2}(r\gamma)^2 f_S^{m2} \vec{\sigma_P}^2 + 2a^{m2}\beta^2 \vec{\sigma_P}^2 + \frac{1}{2} c^{m2} \sigma_a^2 \\
& - 2a^m f_S^m r\gamma\beta \vec{\sigma_P}^2 - c^m f_S^m r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta} + 2a^m c^m \beta \frac{p_a \sigma_a^2}{1 - p_S \beta} \\
0 = {} & -rb^m - r\gamma f_{N^a}^m e_{N^a}^m + c^m \phi_{N^a}^m - 2b^m (p_a + 1)\alpha + \frac{1}{2}(r\gamma)^2 f_{N^a}^{m\,2} \vec{\sigma_P}^2 + \frac{1}{2} c^{m2} \beta^2 \vec{\sigma_P}^2 \\
& + 2b^{m2} \sigma_a^2 - c^m f_{N^a}^m r\gamma\beta \vec{\sigma_P}^2 - 2b^m f_{N^a}^m r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta} + 2b^m c^m \beta \frac{p_a \sigma_a^2}{1 - p_S \beta} \\
0 = {} & -rc^m - r\gamma (f_S^m e_{N^a}^m + f_{N^a}^m e_S^m) + (2a^m \phi_{N^a}^m + c^m \phi_S^m) - \alpha (2b^m p_S + c^m (p_a + 1)) \\
& + (r\gamma)^2 f_S^m f_{N^a}^m \vec{\sigma_P}^2 + 2a^m c^m \beta^2 \vec{\sigma_P}^2 + 2b^m c^m \sigma_a^2 - r\gamma\beta \vec{\sigma_P}^2 (2a^m f_{N^a}^m + c^m f_S^m) \\
& - r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta} (2b^m f_S^m + c^m f_{N^a}^m) + \beta \frac{p_a \sigma_a^2}{1 - p_S \beta} (4a^m b^m + c^{m2}) \\
0 = {} & -rd^m - r\gamma (f_0^m e_S^m + f_S^m e_0^m) + (2a^m \phi_0^m + d^m \phi_S^m) + \alpha (-e^m p_S + (p_0^r - p_0) c^m) \\
& + (r\gamma)^2 \vec{\sigma_P}^2 f_0^m f_S^m + 2a^m d^m \beta^2 \vec{\sigma_P}^2 + c^m e^m \sigma_a^2 - r\gamma\beta \vec{\sigma_P}^2 (2a^m f_0^m + d^m f_S^m) \\
& - r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta} (c^m f_0^m + e^m f_S^m) + \beta \frac{p_a \sigma_a^2}{1 - p_S \beta} (2a^m e^m + c^m d^m) \\
0 = {} & -re^m - r\gamma (f_0^m e_{N^a}^m + f_{N^a}^m e_0^m) + (c^m \phi_0^m + d^m \phi_{N^a}^m) + \alpha (2b^m (p_0^r - p_0) - e^m (p_a + 1)) \\
& + (r\gamma)^2 \vec{\sigma_P}^2 f_0^m f_{N^a}^m + c^m d^m \beta^2 \vec{\sigma_P}^2 + 2b^m e^m \sigma_a^2 - r\gamma\beta \vec{\sigma_P}^2 (c^m f_0^m + d^m f_{N^a}^m) \\
& - r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta} (2b^m f_0^m + e^m f_{N^a}^m) + \beta \frac{p_a \sigma_a^2}{1 - p_S \beta} (c^m e^m + 2b^m d^m) \\
0 = {} & r - \delta - rf^m - r \log (r\gamma) - r\gamma f_0^m e_0^m + d^m \phi_0^m + \alpha e^m (p_0^r - p_0) + \frac{1}{2}(r\gamma)^2 f_0^{m2} \vec{\sigma_P}^2 \\
& + \frac{1}{2}\beta^2 \vec{\sigma_P}^2 (2a^m + d^{m2}) + \frac{1}{2}\sigma_a^2 (2b^m + e^{m2}) - r\gamma\beta d^m f_0^m \vec{\sigma_P}^2 - r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta} e^m f_0^m \\
& + \beta \frac{p_a \sigma_a^2}{1 - p_S \beta} (c^m + d^m e^m)
\end{aligned}
\tag{B.14}
$$

Solving this quadratic equations system, we are able to determine the coefficients, so as to pin down the optimal consumption and portfolio policies for monopolistic traders in terms of state variables S_t and N_t^a. The market clearing condition then implies

$$
\mu_0 (f_0^e + f_S^e S_t + f_{N^a}^e N_t^a) + \mu_1 N_t^a + \mu_2 (f_0^m + f_S^m S_t + f_{N^a}^m N_t^a) = Q
$$

This is true for all S_t and N_t^a, gives us three additional equations:

$$\mu_0 f_0^e + \mu_2 f_0^m - Q = 0 \tag{B.15}$$

$$\mu_0 f_S^e + \mu_2 f_S^m = 0 \tag{B.16}$$

$$\mu_0 f_{N^a}^e + \mu_1 + \mu_2 f_{N^a}^m = 0 \tag{B.17}$$

By these three equations we are able to further pin down the coefficients p_0, p_S, and p_a in the price function. This is the explicit representation of the equilibrium price. ■

B.2.3 Monopolistic Traders' Optimization Problem CASE (b)

In this case the monopolistic traders have both market power and correct beliefs. He takes the market clearing condition into consideration. The monopolistic traders have infinite horizon at $[0, \infty)$. He tries to maximize the expected utility on the initial time 0

$$\mathbb{E}_0^m \left[- \int_0^\infty \frac{e^{-\delta t - \gamma C_t^m}}{\gamma} dt \right]$$

subject to the budget constraint and state processes

$$\begin{aligned} dW_t^m &= (rW_t^m - C_t^m)dt + N_t^m[(D_t - rP_t)dt + dP_t] \\ dS_t &= -\beta S_t dt + \beta dP_t \\ dN_t^a &= \alpha[(P_t^r - P_t) - N_t^a]dt + \sigma_a dZ_t^a \end{aligned}$$

where P_t is the true price process of risky asset and N_t^a is the arbitrageurs' true demand.

In order to characterize investment opportunity in this economy, again we consider the instantaneous excess return to one share of stock: $dQ_t^m = (D_t - rP_t)dt + dP_t$, which can also be thought as the return on a zero-wealth portfolio long one share of stock fully financed by borrowing at the risk-free rate.

$$
\begin{aligned}
dP_t &= p_S dS_t + \frac{1}{r}D_t + p_a dN_t^a \\
&= -p_S\beta S_t dt + p_S\beta dP_t + \frac{1}{r}g_D dt + \frac{1}{r}\sigma_D dZ_t^D + p_a\alpha(P_t^r - P_t - N_t^a)dt + p_a\sigma_a dZ_t^a \\
&= \frac{1}{1-p_S\beta}\left[\frac{g_D}{r} + p_a\alpha(p_0^r - p_0) - (p_S\beta + p_a p_S\alpha)S_t - p_a(p_a+1)\alpha N_t^a\right]dt \\
&\quad + \frac{1}{1-p_S\beta}\left(\frac{\sigma_D}{r}dZ_t^D + p_a\sigma_a dZ_t^a\right)
\end{aligned}
$$

Since monopolistic traders have market power, by market clearing condition and extrapolators' optimal portfolio strategy, we have

$$
\mu_0 N_t^e = \mu_0(f_0 + f_S S_t + f_{N^a}N_t^a) = Q - \mu_1 N_t^a - \mu_2 N_t^m
$$

Plug in the representations of f_0, f_S, f_{N^a}, and the price function, the above market clearing condition can be transformed into

$$
\begin{aligned}
N_t^e &= \frac{1}{r\gamma\vec{\sigma_P}^2}\left[(\lambda_0 + \beta d^e\vec{\sigma_P}^2 + e^e\frac{p_a\sigma_a^2}{1-p_S\beta}) + (\lambda_1 + 2a^e\beta\vec{\sigma_P}^2 + c^e\frac{p_a\sigma_a^2}{1-p_S\beta})S_t\right. \\
&\quad \left. +(c^e\beta\vec{\sigma_P}^2 + 2b^e\frac{p_a\sigma_a^2}{1-p_S\beta})N_t^a\right] - \frac{1}{\gamma\vec{\sigma_P}^2}(p_0 + p_S S_t + p_a N_t^a) \\
&= \frac{1}{r\gamma\vec{\sigma_P}^2}\left[(\lambda_0 + \beta d^e\vec{\sigma_P}^2 + e^e\frac{p_a\sigma_a^2}{1-p_S\beta}) + (\lambda_1 + 2a^e\beta\vec{\sigma_P}^2\right. \\
&\quad \left. +c^e\frac{p_a\sigma_a^2}{1-p_S\beta})S_t + (c^e\beta\vec{\sigma_P}^2 + 2b^e\frac{p_a\sigma_a^2}{1-p_S\beta})N_t^a\right] + \frac{1}{r\gamma\vec{\sigma_P}^2}(D_t - rP_t)
\end{aligned}
$$

$$
\begin{aligned}
dQ_t^m &= (D_t - rP_t)dt + dP_t \\
&= [(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a + e_{N^m}^m N_t^m)]dt + \vec{\sigma_P}d\vec{Z_t}
\end{aligned}
$$

Where $\vec{\sigma_P}d\vec{Z_t}$ is specified before and e_0^m, e_S^m, $e_{N^a}^m$, and $e_{N^m}^m$ has the following form:

$$
\begin{aligned}
e_0^m &= \frac{r\gamma\vec{\sigma_P}^2 Q}{\mu_0} - (\lambda_0 + \beta d^e \vec{\sigma_P}^2 + e^e \frac{p_a \sigma_a^2}{1 - p_S \beta}) + \frac{g_D/r + p_a \alpha (p_0^r - p_0)}{(1 - p_S \beta)} \\
e_S^m &= -(\lambda_1 + 2a^e \beta \vec{\sigma_P}^2 + c^e \frac{p_a \sigma_a^2}{1 - p_S \beta}) - \frac{p_S \beta + p_a p_S \alpha}{1 - p_S \beta} \\
e_{N^a}^m &= -\frac{\mu_1 r\gamma \vec{\sigma_P}^2}{\mu_0} - (c^e \beta \vec{\sigma_P}^2 + 2b^e \frac{p_a \sigma_a^2}{1 - p_S \beta}) - \frac{p_a (p_a + 1)\alpha}{1 - p_S \beta} \\
e_{N^m}^m &= -\frac{\mu_2 r\gamma \vec{\sigma_P}^2}{\mu_0}
\end{aligned}
$$

The monopolistic traders' optimization problem now becomes

$$
\mathbb{E}_0^m \left[-\int_0^\infty \frac{e^{-\delta t - \gamma C_t^m}}{\gamma} dt \right]
$$

subject to the budget constraint

$$
\begin{aligned}
dW_t^m &= (rW_t^m - C_t^m)dt + N_t^m dQ_t^m \\
&= (rW_t^m - C_t^m)dt + N_t^m (e_0^m + e_S^m S_t + e_{N^a}^m N_t^a + e_{N^m}^m N_t^m)dt + N_t^m \vec{\sigma_P} d\vec{Z_t} \\
dS_t &= \frac{\beta}{1 - p_S \beta} \left[\frac{g_D}{r} + p_a \alpha (p_0^r - p_0) - (1 + p_a p_S \alpha) S_t - p_a (p_a + 1) \alpha N_t^a \right] dt + \beta \vec{\sigma_P} d\vec{Z_t} \\
&= (\phi_0^m + \phi_S^m S_t + \phi_{N^a}^m N_t^a)dt + \beta \vec{\sigma_P} d\vec{Z_t} \\
dN_t^a &= \alpha[(p_0^r - p_0) - p_S S_t - (p_a + 1)N_t^a]dt + \sigma_a dZ_t^a
\end{aligned}
$$

where

$$
\begin{aligned}
\phi_0^m &= \frac{\beta}{1 - p_S \beta} \frac{g_D + \alpha r p_a (p_0^r - p_0)}{r} \\
\phi_S^m &= -\frac{\beta}{1 - p_S \beta}(1 + p_a p_S \alpha) \\
\phi_{N^a}^m &= -\frac{\beta}{1 - p_S \beta} p_a (p_a + 1)\alpha
\end{aligned}
$$

Assuming the derived value function for the monopolistic traders is

$$
J_t^m(W_t^m; S_t, N_t^a; t) \equiv \max_{\{C_s^m, N_s^m\}_{s \ge t}} \mathbb{E}_t^m \left[-\int_t^\infty \frac{e^{-\delta s - \gamma C_s^m}}{\gamma} ds \right]
$$

Using Ito's lemma and HJB equation, we are able to obtain the following PDE:

$$
\begin{aligned}
0 = & -\frac{e^{-\delta t-\gamma C_t^m}}{\gamma} - \delta J^m + J_W^m dW^m + J_S^m dS + J_{N^a}^m dN^a \\
& + \frac{1}{2} J_{WW}^m dW^m dW^m + \frac{1}{2} J_{SS}^m dS\, dS + \frac{1}{2} J_{N^a N^a}^m dN^a dN^a \\
& + J_{WS}^m dW^m dS + J_{WN^a}^m dW^m dN^a + J_{SN^a}^m dS\, dN^a
\end{aligned}
\tag{B.18}
$$

We conjecture, and later verify, that the value function J_t^m has the following form:

$$
J_t^m = -e^{-\delta t - r\gamma W^m + a^m S^2 + b^m (N^a)^2 + c^m S N^a + d^m S + e^m N^a + f^m}
$$

The we can calculate the partial differentials:

$$
\begin{aligned}
J_W^m &= -r\gamma J^m \\
J_S^m &= (2a^m S + c^m N^a + d^m) J^m \\
J_{N^a}^m &= (2b^m N^a + c^m S + e^m) J^m \\
J_{WW}^m &= (r\gamma)^2 J^m \\
J_{SS}^m &= [2a^m + (2a^m S + c^m N^a + d^m)^2] J^m \\
J_{N^a N^a}^m &= [2b^m + (2b^m N^a + c^m S + e^m)^2] J^m \\
J_{WS}^m &= -r\gamma (2a^m S + c^m N^a + d^m) J^m \\
J_{WN^a}^m &= -r\gamma (2b^m N^a + c^m S + e^m) J^m \\
J_{SN^a}^m &= [c^m + (2a^m S + c^m N^a + d^m)(2b^m N^a + c^m S + e^m)] J^m
\end{aligned}
$$

Thus equation B.18 can be transformed into

$$
\begin{aligned}
0 = {} & r - \delta \\
& - r\left[a^m S_t^2 + b^m (N_t^a)^2 + c^m S_t N_t^a + d^m S_t + e^m N_t^a + f^m + \log(r\gamma)\right] \\
& - r\gamma N_t^m (e_0^m + e_S^m S_t + e_{N^a}^m N_t^a + e_{N^m}^m N_t^m) \\
& + (2a^m S_t + c^m N_t^a + d^m)(\phi_0^m + \phi_S^m S_t + \phi_{N^a}^m N_t^a) \\
& + \alpha(2b^m N_t^a + c^m S_t + e^m)((p_0^r - p_0) - p_S S_t - (p_a + 1)N_t^a) \\
& + \frac{1}{2}(r\gamma)^2 \vec{\sigma_P}^2 (N_t^m)^2 + \frac{1}{2}[2a^m + (2a^m S_t + c^m N_t^a + d^m)^2]\beta^2 \vec{\sigma_P}^2 \\
& + \frac{1}{2}\sigma_a^2[2b^m + (2b^m N_t^a + c^m S_t + e^m)^2] \\
& - r\gamma\beta(2a^m S_t + c^m N_t^a + d^m)\vec{\sigma_P}^2 N_t^m - r\gamma(2b^m N_t^a + c^m S_t + e^m)\frac{p_a \sigma_a^2}{1 - p_S \beta} N_t^m \\
& + [c^m + (2a^m S_t + c^m N_t^a + d^m)(2b^m N_t^a + c^m S_t + e^m)]\beta \frac{p_a \sigma_a^2}{1 - p_S \beta}
\end{aligned}
\tag{B.19}
$$

Take first order condition with respect to N_t^e, we obtain the optimal portfolio strategy

$$
\begin{aligned}
0 = {} & -r\gamma(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a) - 2r\gamma e_{N^m}^m N_t^m + (r\gamma)^2 \vec{\sigma_P}^2 N_t^m - r\gamma\beta(2a^m S_t + c^m N_t^a + d^m)\vec{\sigma_P}^2 \\
& - r\gamma(2b^m N_t^a + c^m S_t + e^m)\frac{p_a \sigma_a^2}{1 - p_S \beta}
\end{aligned}
$$

Rearrange the above equation, we can represent N_t^m in terms of a linear combination of our state variables S_t and N_t^a as

$$
N_t^m = f_0^m + f_S^m S_t + f_{N^a}^m N_t^a \tag{B.20}
$$

where

$$
\begin{aligned}
f_0^m &= \frac{e_0^m + \beta d^m \vec{\sigma_P}^2 + e^m \frac{p_a \sigma_a^2}{1 - p_S \beta}}{r\gamma \vec{\sigma_P}^2 - 2e_{N^m}^m} \\
f_S^m &= \frac{e_S^m + 2a^m \beta \vec{\sigma_P}^2 + c^m \frac{p_a \sigma_a^2}{1 - p_S \beta}}{r\gamma \vec{\sigma_P}^2 - 2e_{N^m}^m} \\
f_{N^a}^m &= \frac{e_{N^a}^m + c^m \beta \vec{\sigma_P}^2 + 2b^m \frac{p_a \sigma_a^2}{1 - p_S \beta}}{r\gamma \vec{\sigma_P}^2 - 2e_{N^m}^m}
\end{aligned}
$$

Plug the optimal portfolio strategy back into equation B.19, we obtain the following quadratic equation in S and N^a

$$
\begin{aligned}
0 = {} & r - \delta \\
& - r[a^m S_t^2 + b^m (N_t^a)^2 + c^m S_t N_t^a + d^m S_t + e^m N_t^a + f^m + \log(r\gamma)] \\
& - r\gamma(f_0^m + f_S^m S_t + f_{N^a}^m N_t^a)(e_0^m + e_S^m S_t + e_{N^a}^m N_t^a) - r\gamma e_{N^m}^m (f_0^m + f_S^m S_t + f_{N^a}^m N_t^a)^2 \\
& + (2a^m S_t + c^m N_t^a + d^m)(\phi_0^m + \phi_S^m S_t + \phi_{N^a}^m N_t^a) \\
& + \alpha(2b^m N_t^a + c^m S_t + e^m)((p_0^r - p_0) - p_S S_t - (p_a + 1) N_t^a) \\
& + \frac{1}{2}(r\gamma)^2 \vec{\sigma_P}^2 (f_0^m + f_S^m S_t + f_{N^a}^m N_t^a)^2 + \frac{1}{2}[2a^m + (2a^m S_t + c^m N_t^a + d^m)^2]\beta^2 \vec{\sigma_P}^2 \\
& + \frac{1}{2}\sigma_a^2 [2b^m + (2b^m N_t^a + c^m S_t + e^m)^2] \\
& - r\gamma\beta\vec{\sigma_P}^2 (2a^m S_t + c^m N_t^a + d^m)(f_0^m + f_S^m S_t + f_{N^a}^m N_t^a) \\
& - r\gamma \frac{p_a \sigma_a^2}{1 - p_S \beta}(2b^m N_t^a + c^m S_t + e^m)(f_0^m + f_S^m S_t + f_{N^a}^m N_t^a) \\
& + \beta \frac{p_a \sigma_a^2}{1 - p_S \beta}[c^m + (2a^m S_t + c^m N_t^a + d^m)(2b^m N_t^a + c^m S_t + e^m)]
\end{aligned}
\tag{B.21}
$$

The above equation is valid for any value of S_t and N_t^a, which implies the coefficients of variables should be zero. That is

$$
\begin{aligned}
0 &= -ra^m - r\gamma f_S^m e_S^m + 2a^m \phi_S^m - c^m p_S \alpha + \frac{1}{2}(r\gamma)^2 f_S^{m2} \vec{\sigma_P}^2 + 2a^{m2}\beta^2 \vec{\sigma_P}^2 + \frac{1}{2}c^{m2}\sigma_a^2 \\
&\quad - 2a^m f_S^m r\gamma\beta \vec{\sigma_P}^2 - c^m f_S^m r\gamma \frac{p_a \sigma_a^2}{1-p_S\beta} + 2a^m c^m \beta \frac{p_a\sigma_a^2}{1-p_S\beta} - r\gamma e_{N^m}^m (f_S^m)^2 \\
0 &= -rb^m - r\gamma f_{N^a}^m e_{N^a}^m + c^m \phi_{N^a}^m - 2b^m(p_a+1)\alpha + \frac{1}{2}(r\gamma)^2 f_{N^a}^{m\,2} \vec{\sigma_P}^2 + \frac{1}{2}c^{m2}\beta^2\vec{\sigma_P}^2 \\
&\quad + 2b^{m2}\sigma_a^2 - c^m f_{N^a}^m r\gamma\beta\vec{\sigma_P}^2 - 2b^m f_{N^a}^m r\gamma \frac{p_a\sigma_a^2}{1-p_S\beta} + 2b^m c^m \beta \frac{p_a\sigma_a^2}{1-p_S\beta} - r\gamma e_{N^m}^m (f_{N^a}^m)^2 \\
0 &= -rc^m - r\gamma(f_S^m e_{N^a}^m + f_{N^a}^m e_S^m) + (2a^m\phi_{N^a}^m + c^m \phi_S^m) - \alpha(2b^m p_S + c^m(p_a+1)) \\
&\quad + (r\gamma)^2 f_S^m f_{N^a}^m \vec{\sigma_P}^2 + 2a^m c^m \beta^2 \vec{\sigma_P}^2 + 2b^m c^m \sigma_a^2 - r\gamma\beta\vec{\sigma_P}^2(2a^m f_{N^a}^m + c^m f_S^m) \\
&\quad - r\gamma \frac{p_a\sigma_a^2}{1-p_S\beta}(2b^m f_S^m + c^m f_{N^a}^m) + \beta\frac{p_a\sigma_a^2}{1-p_S\beta}(4a^m b^m + c^{m2}) - 2r\gamma e_{N^m}^m f_S^m f_{N^a}^m \\
0 &= -rd^m - r\gamma(f_0^m e_S^m + f_S^m e_0^m) + (2a^m \phi_0^m + d^m\phi_S^m) + \alpha(-e^m p_S + (p_0^r - p_0)c^m) \\
&\quad + (r\gamma)^2 \vec{\sigma_P}^2 f_0^m f_S^m + 2a^m d^m \beta^2 \vec{\sigma_P}^2 + c^m e^m \sigma_a^2 - r\gamma\beta\vec{\sigma_P}^2(2a^m f_0^m + d^m f_S^m) \\
&\quad - r\gamma\frac{p_a\sigma_a^2}{1-p_S\beta}(c^m f_0^m + e^m f_S^m) + \beta\frac{p_a\sigma_a^2}{1-p_S\beta}(2a^m e^m + c^m d^m) - 2r\gamma e_{N^m}^m f_0^m f_S^m \\
0 &= -re^m - r\gamma(f_0^m e_{N^a}^m + f_{N^a}^m e_0^m) + (c^m\phi_0^m + d^m\phi_{N^a}^m) + \alpha(2b^m(p_0^r - p_0) - e^m(p_a+1)) \\
&\quad + (r\gamma)^2\vec{\sigma_P}^2 f_0^m f_{N^a}^m + c^m d^m \beta^2 \vec{\sigma_P}^2 + 2b^m e^m \sigma_a^2 - r\gamma\beta\vec{\sigma_P}^2(c^m f_0^m + d^m f_{N^a}^m) \\
&\quad - r\gamma\frac{p_a\sigma_a^2}{1-p_S\beta}(2b^m f_0^m + e^m f_{N^a}^m) + \beta\frac{p_a\sigma_a^2}{1-p_S\beta}(c^m e^m + 2b^m d^m) - 2r\gamma e_{N^m}^m f_0^m f_{N^a}^m \\
0 &= r - \delta - rf^m - r\log(r\gamma) - r\gamma f_0^m e_0^m + d^m\phi_0^m + \alpha e^m (p_0^r - p_0) \\
&\quad + \frac{1}{2}(r\gamma)^2 f_0^{m2}\vec{\sigma_P}^2 + \frac{1}{2}\beta^2\vec{\sigma_P}^2(2a^m + d^{m2}) + \frac{1}{2}\sigma_a^2(2b^m + e^{m2}) - r\gamma\beta d^m f_0^m \vec{\sigma_P}^2 \\
&\quad - r\gamma\frac{p_a\sigma_a^2}{1-p_S\beta} e^m f_0^m + \beta\frac{p_a\sigma_a^2}{1-p_S\beta}(c^m + d^m e^m) - r\gamma e_{N^m}^m (f_0^m)^2
\end{aligned}
\tag{B.22}
$$

Solving this quadratic equations system, we are able to determine the coefficients, so as to pin down the optimal consumption and portfolio policies for monopolistic traders in terms of state variables S_t and N_t^a. The market clearing condition then implies

$$
\mu_0(f_0^e + f_S^e S_t + f_{N^a}^e N_t^a) + \mu_1 N_t^a + \mu_2(f_0^m + f_S^m S_t + f_{N^a}^m N_t^a) = Q
$$

This is true for all S_t and N_t^a, gives us three additional equations:

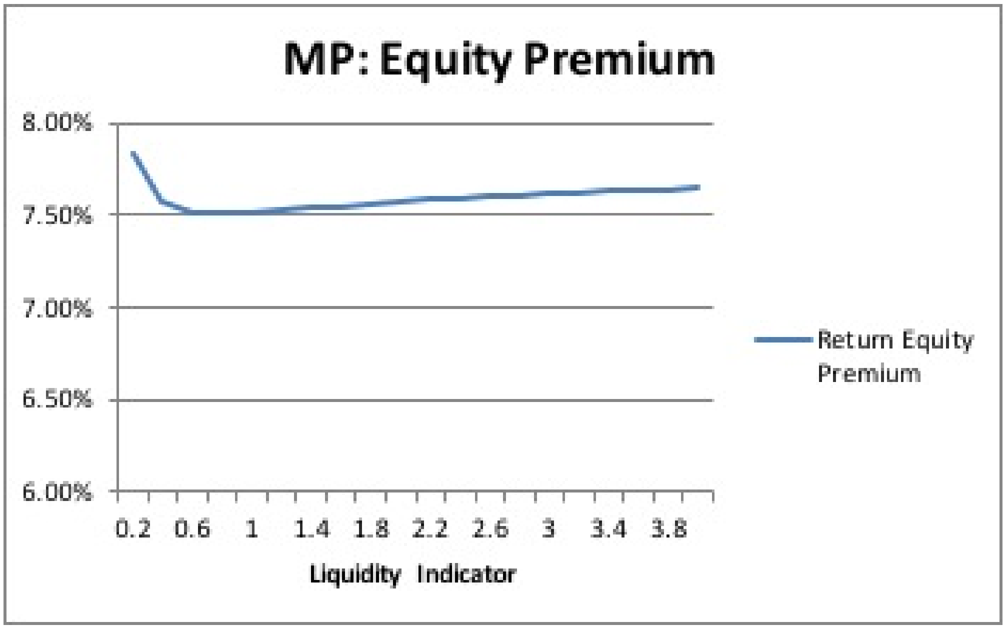
$$\mu_0 f_0^e + \mu_2 f_0^m - Q = 0 \quad \text{(B.23)}$$

$$\mu_0 f_S^e + \mu_2 f_S^m = 0 \quad \text{(B.24)}$$

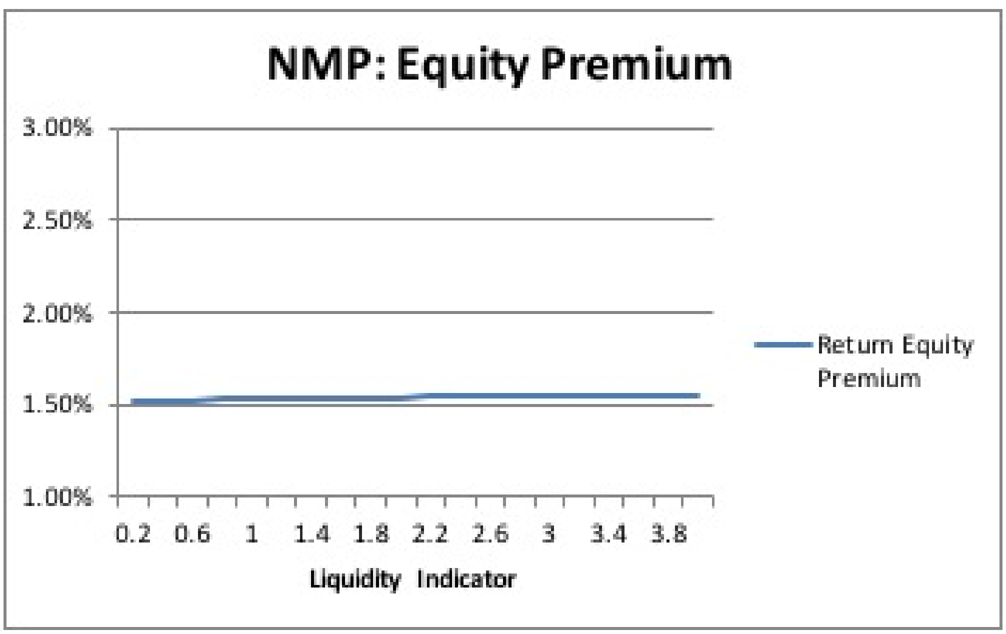
$$\mu_0 f_{N^a}^e + \mu_1 + \mu_2 f_{N^a}^m = 0 \quad \text{(B.25)}$$

By these three equations we are able to further pin down the coefficients p_0, p_S, and p_a in the price function. This is the explicit representation of the equilibrium price. ■

B.3 The Limits of Arbitrage

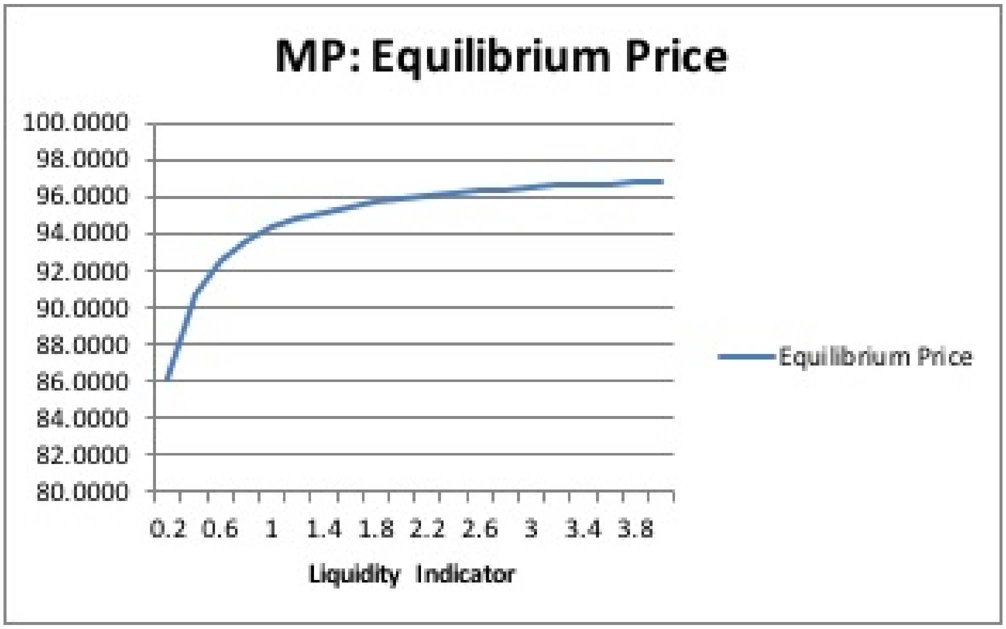


(a) Market Power

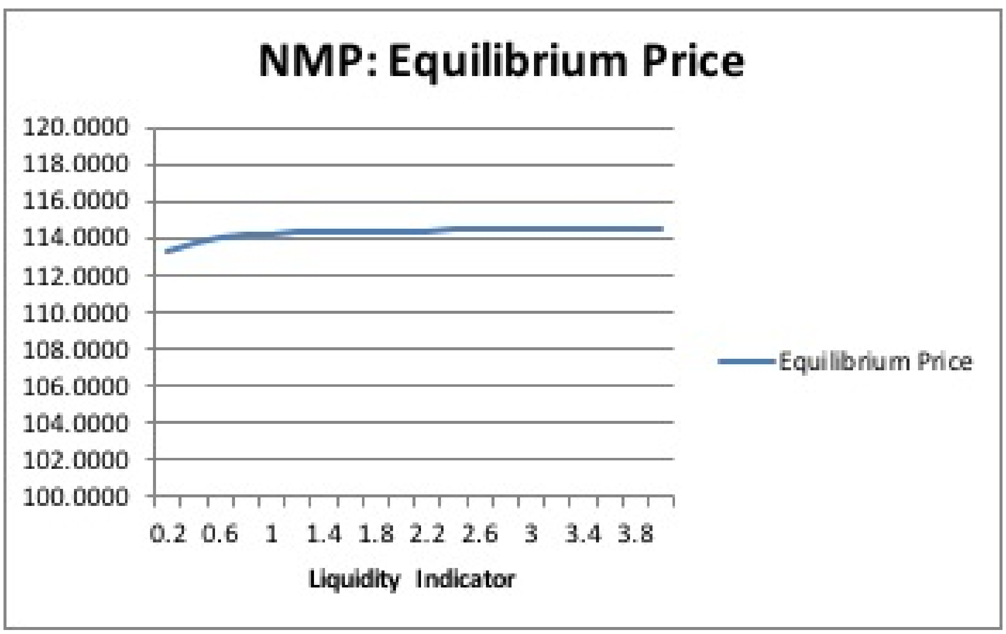


(b) No Market Power

Figure B.1: Liquidity: excess return

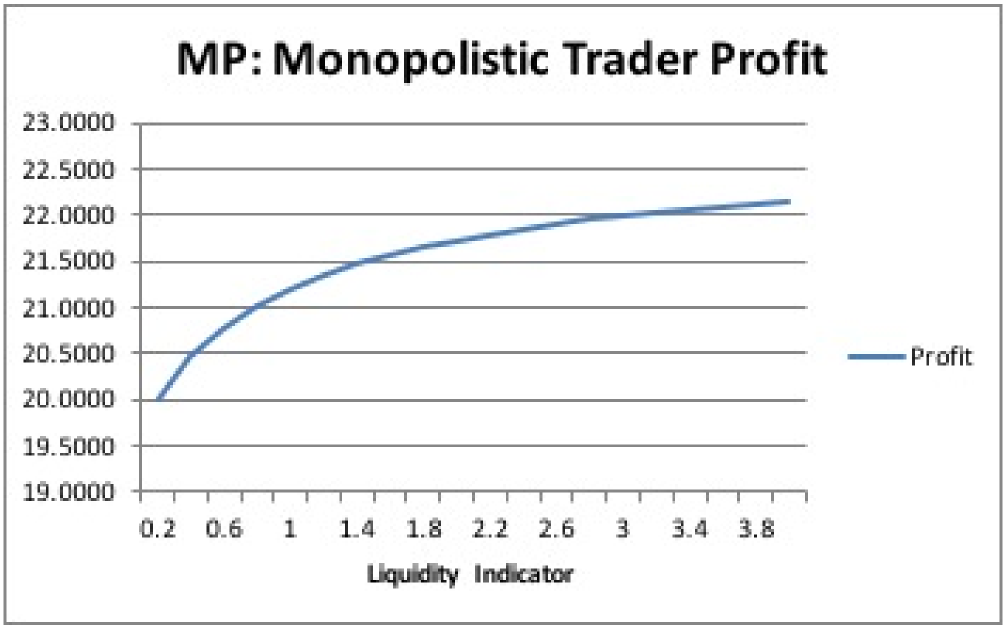


(a) Market Power

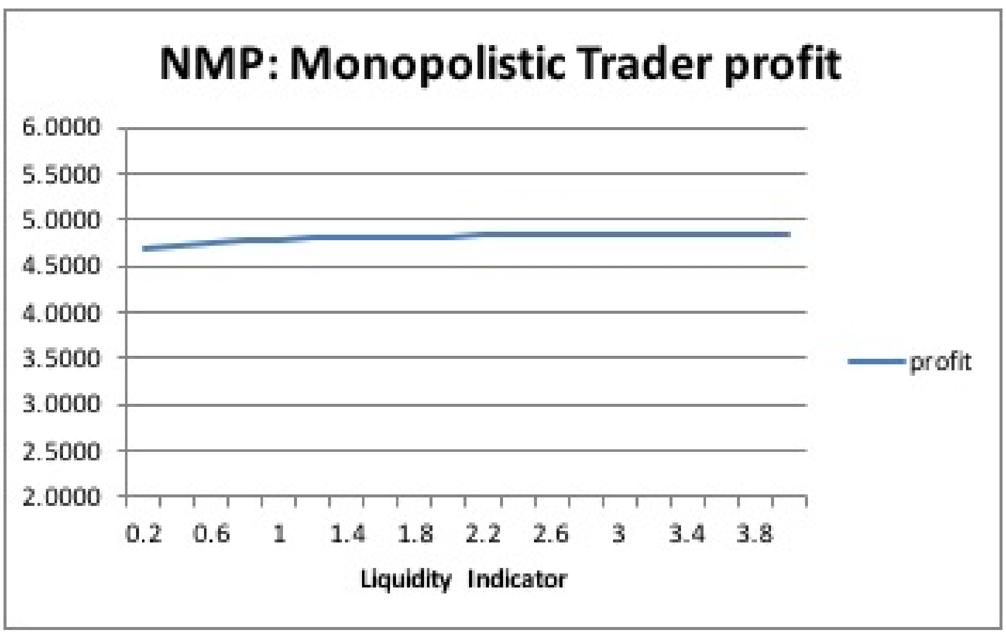


(b) No Market Power

Figure B.2: Liquidity: equilibrium price

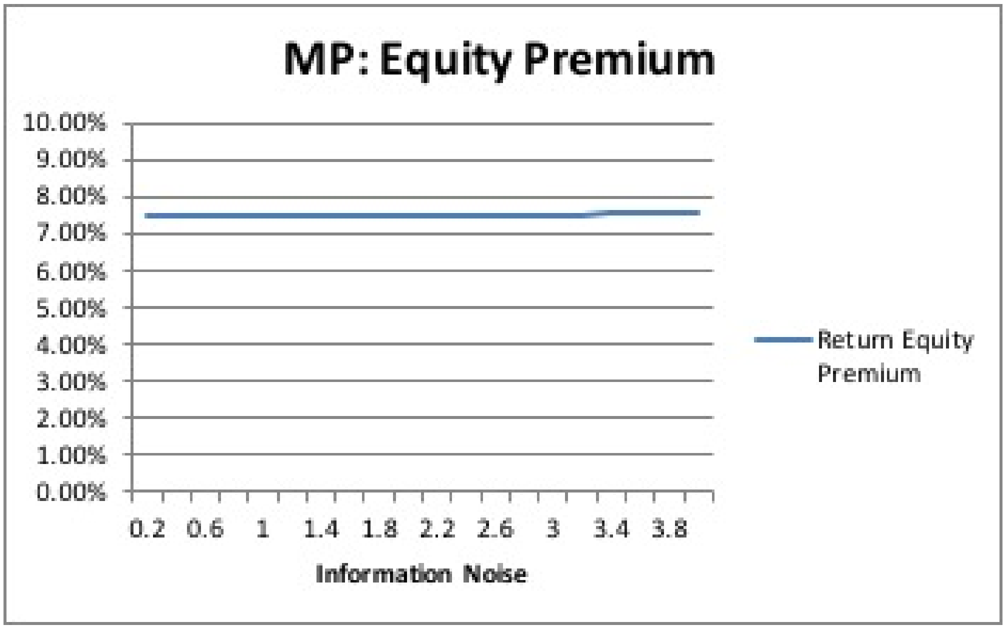


(a) Market Power

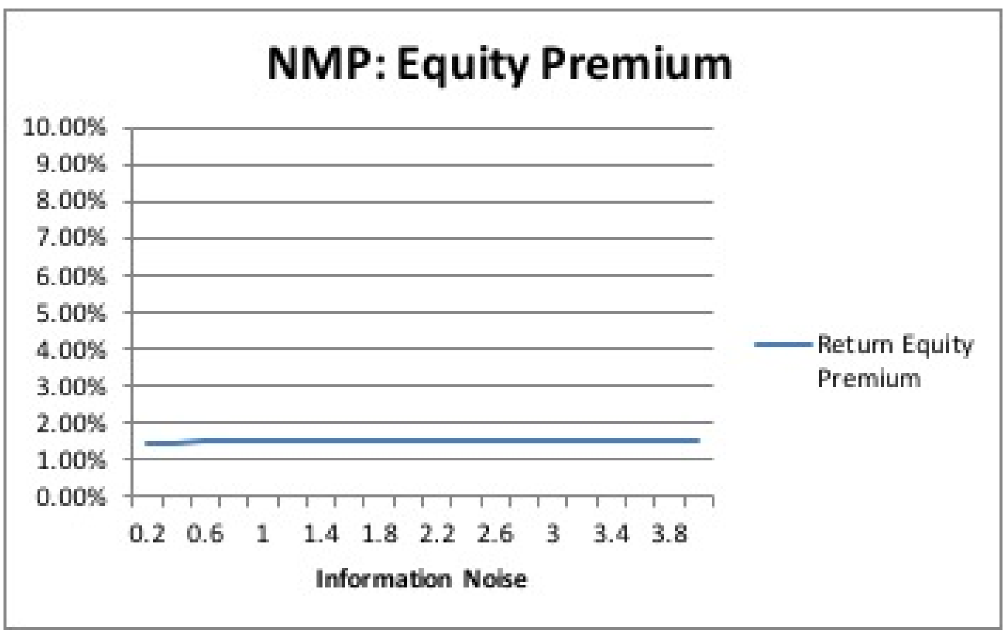


(b) No Market Power

Figure B.3: Liquidity: profit

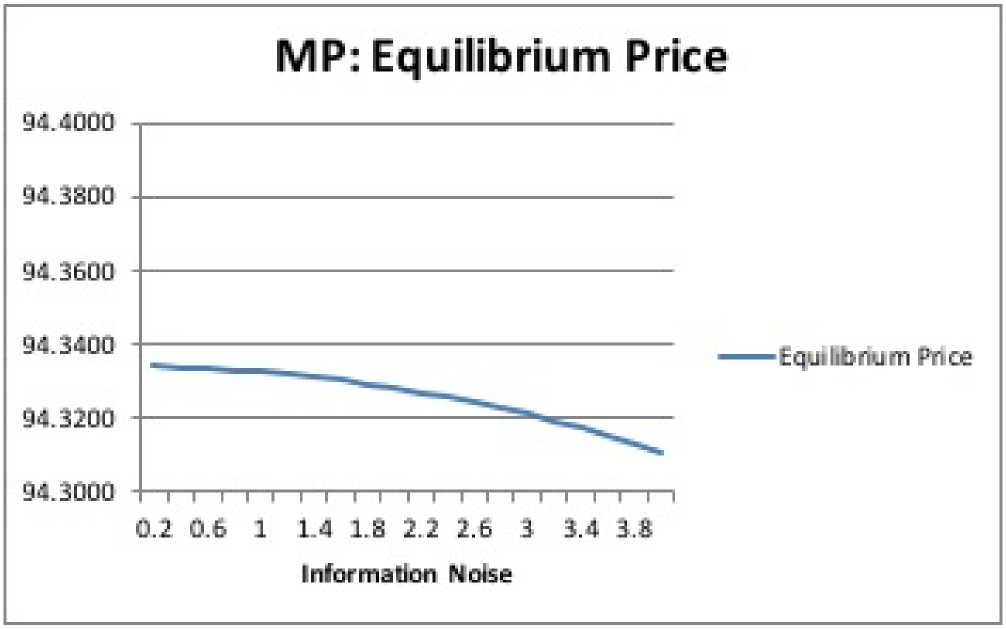


(a) Market Power

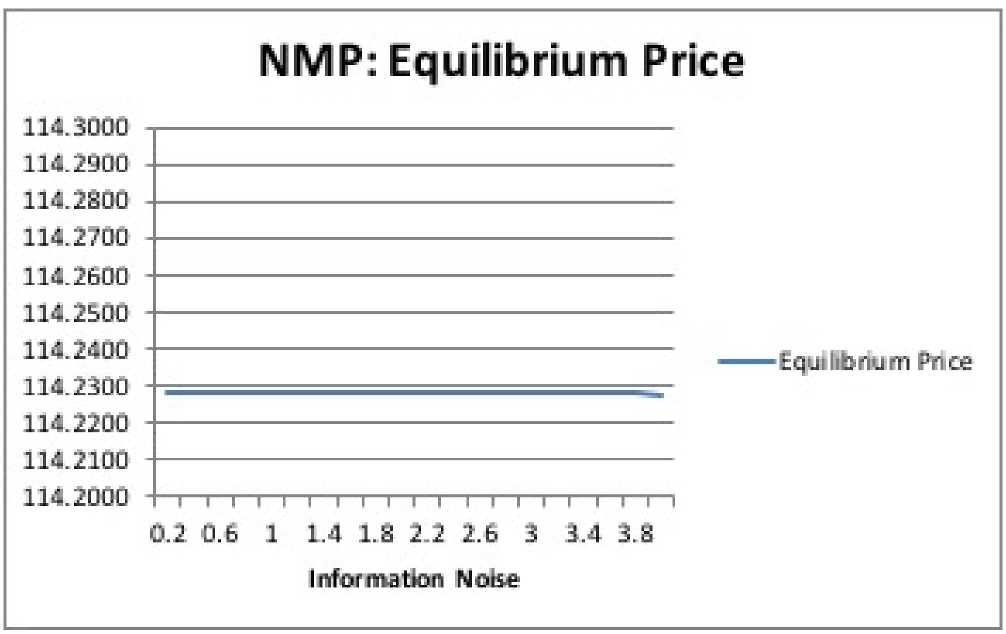


(b) No Market Power

Figure B.4: Information: excess return

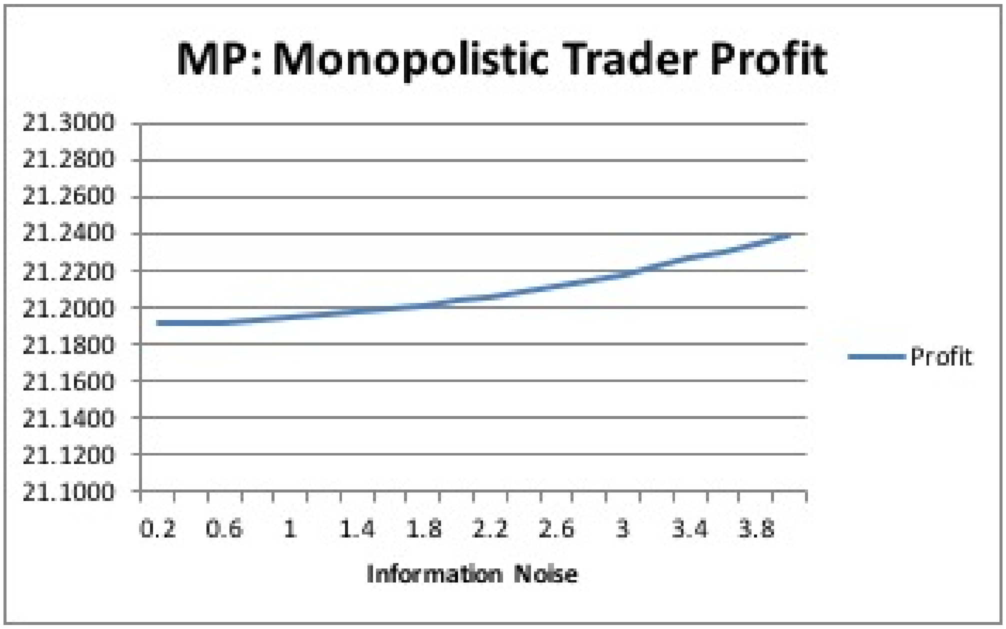


(a) Market Power

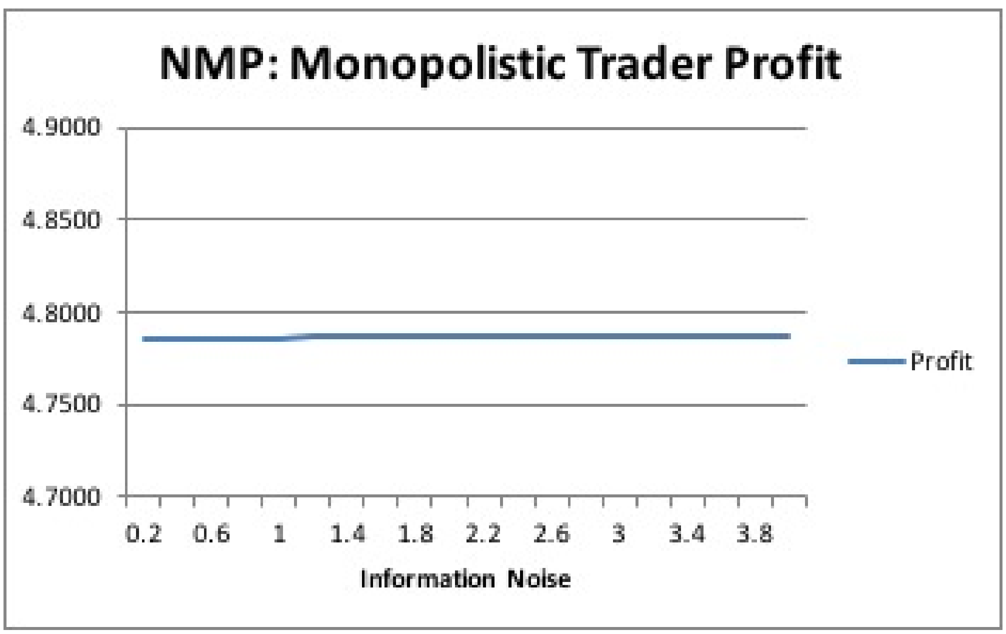


(b) No Market Power

Figure B.5: Information: equilibrium price



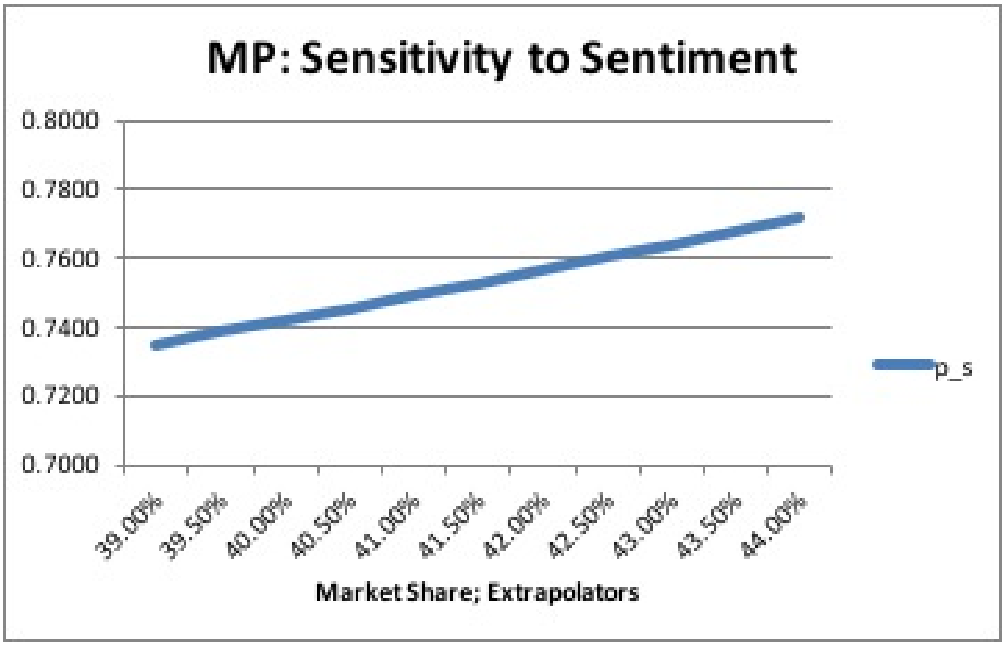
(a) Market Power



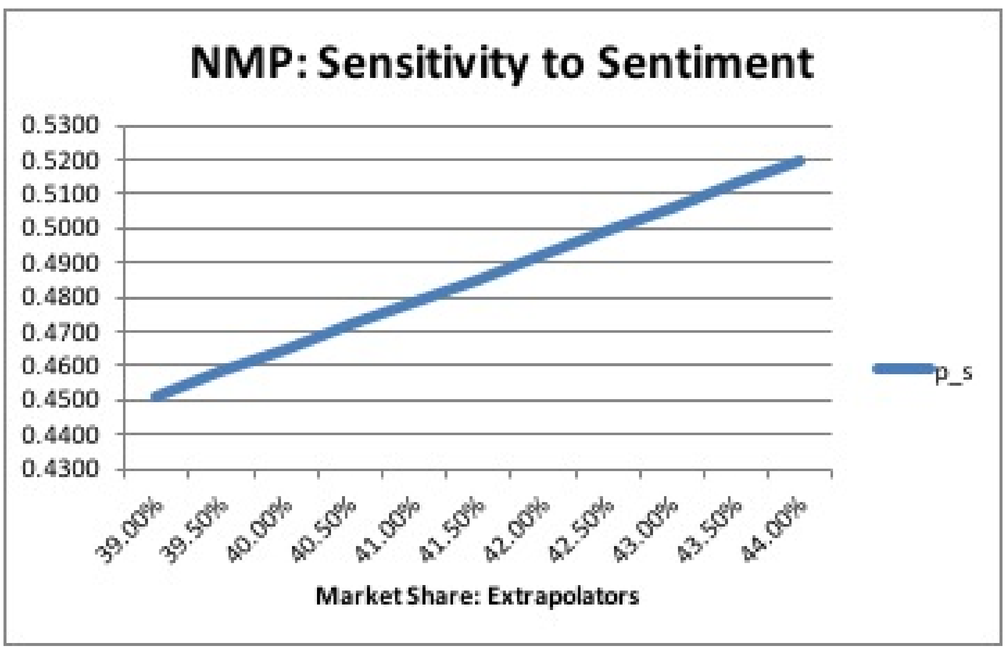
(b) No Market Power

Figure B.6: Information: profit

B.4 Extrapolators Market Share

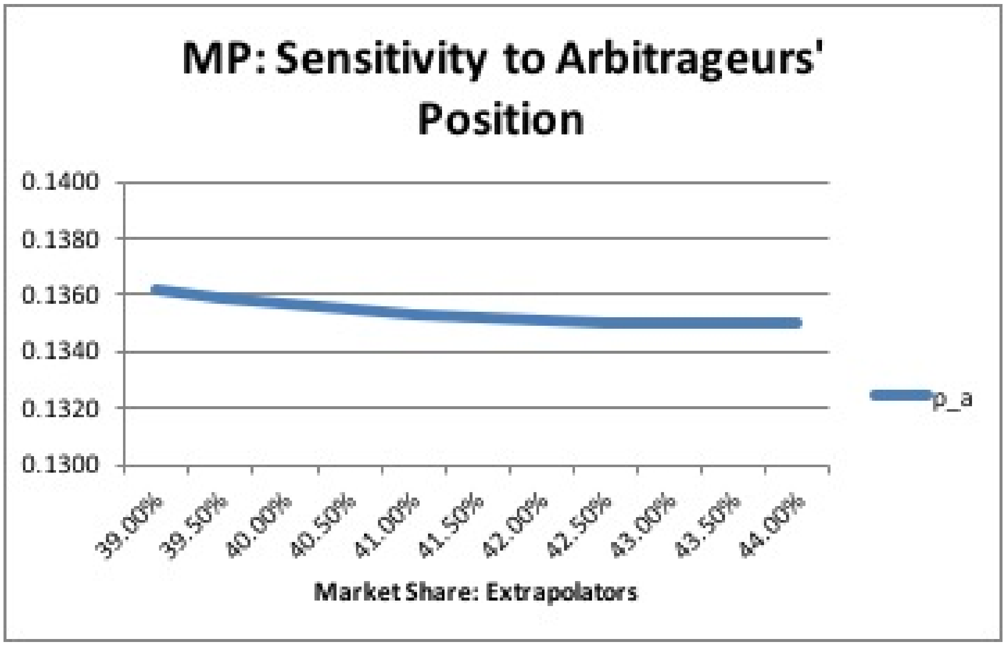


(a) Market Power

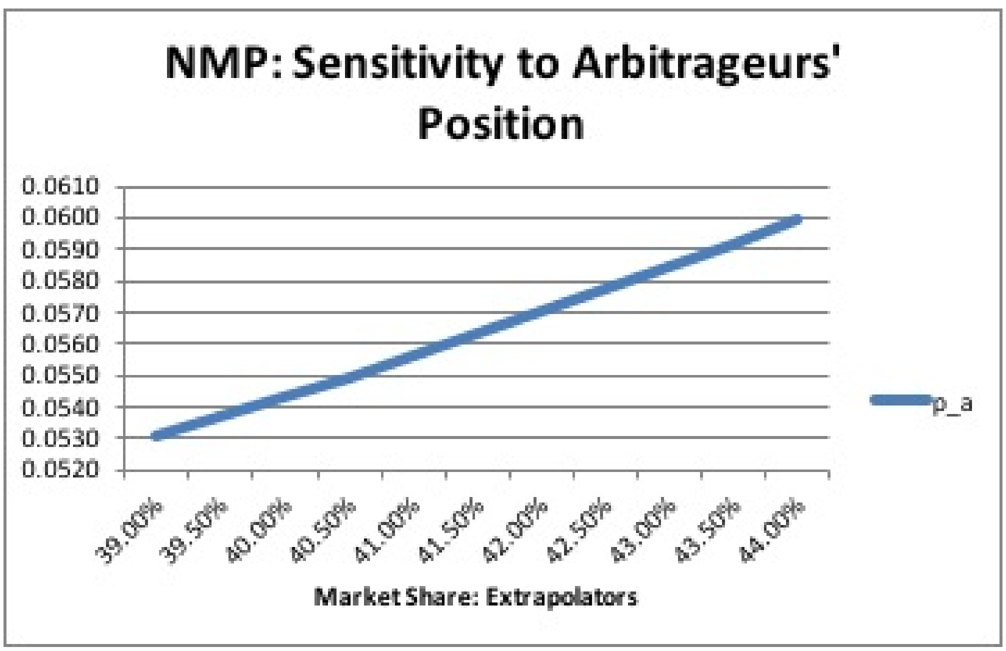


(b) No Market Power

Figure B.7: Extrapolators market share: sensitivity to sentiment

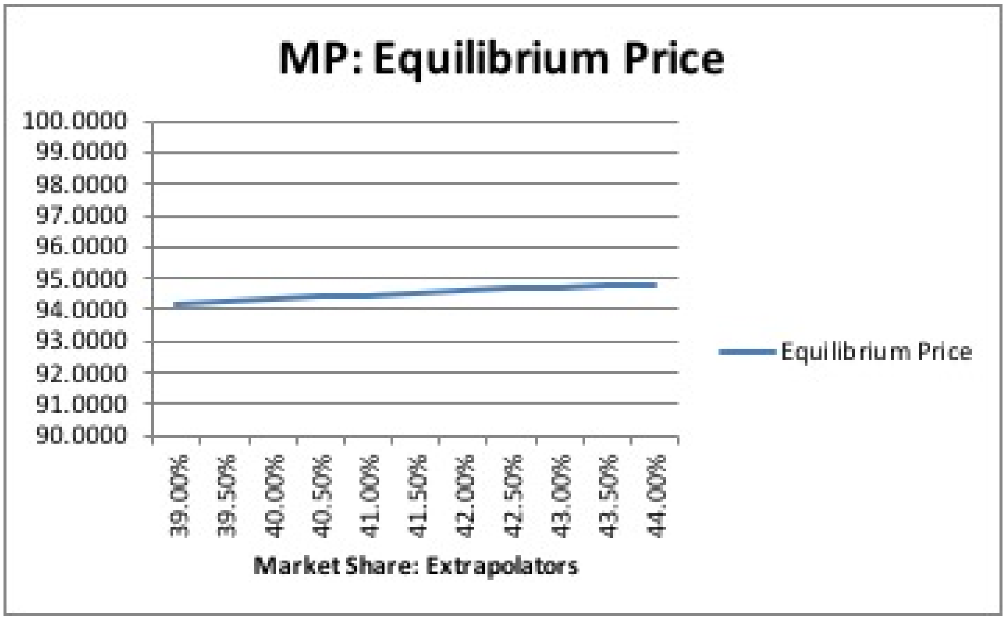


(a) Market Power

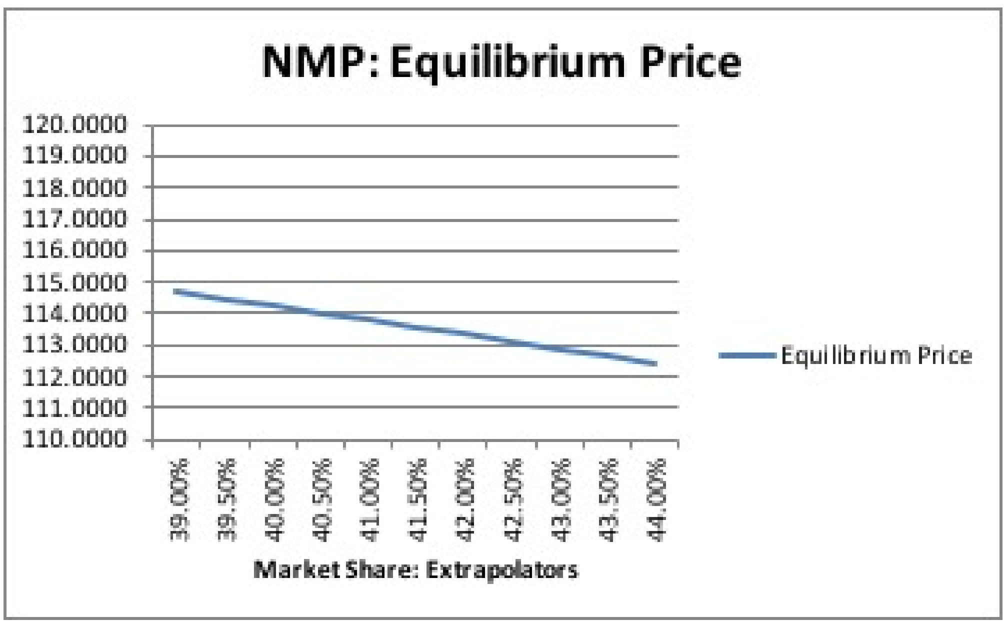


(b) No Market Power

Figure B.8: Extrapolators market share: sensitivity to arbitrageurs

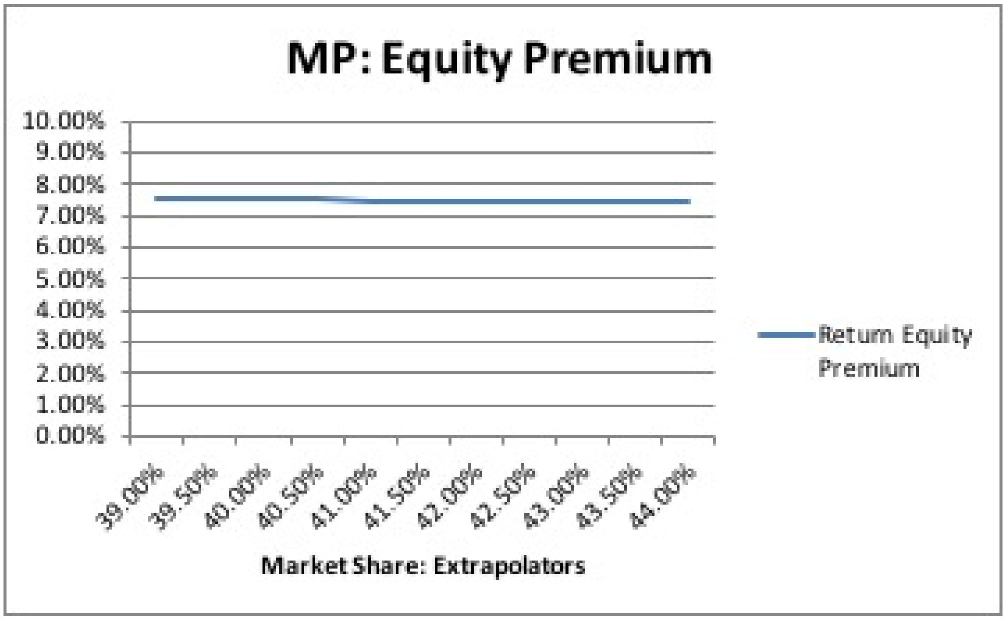


(a) Market Power

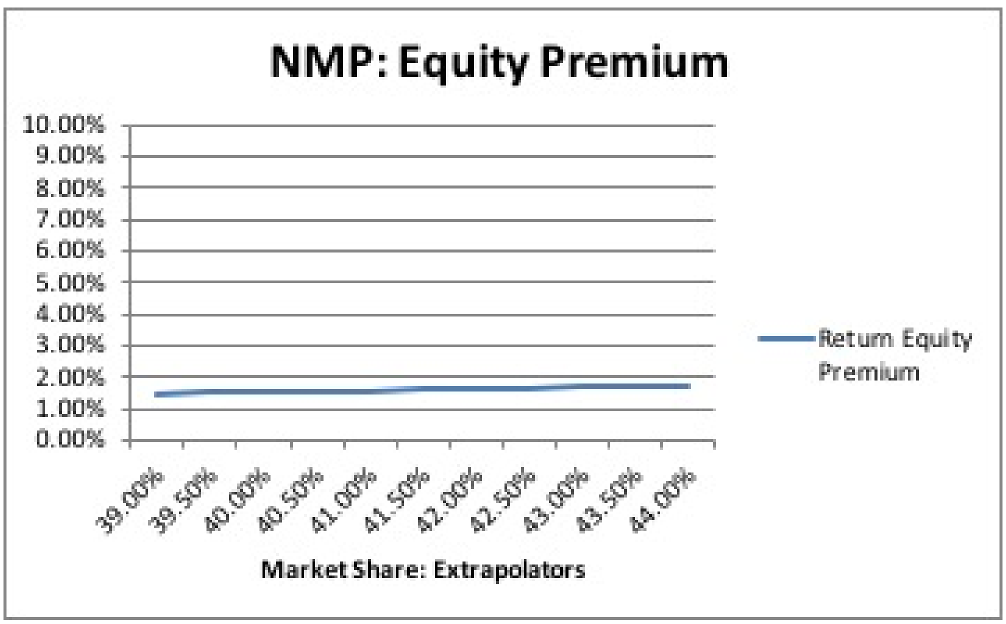


(b) No Market Power

Figure B.9: Extrapolators market share: equilibrium price

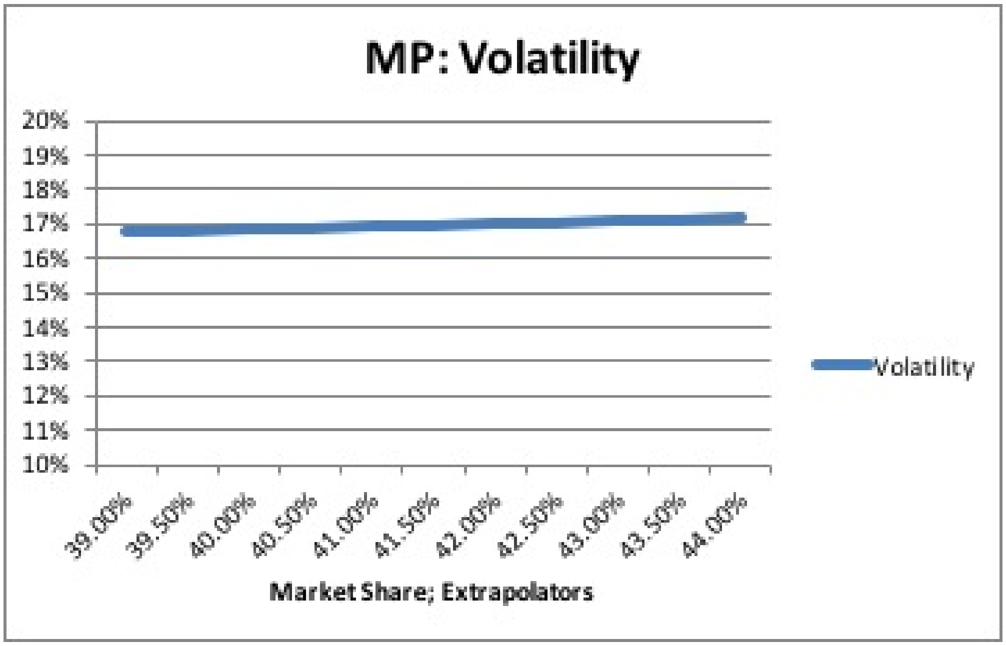


(a) Market Power

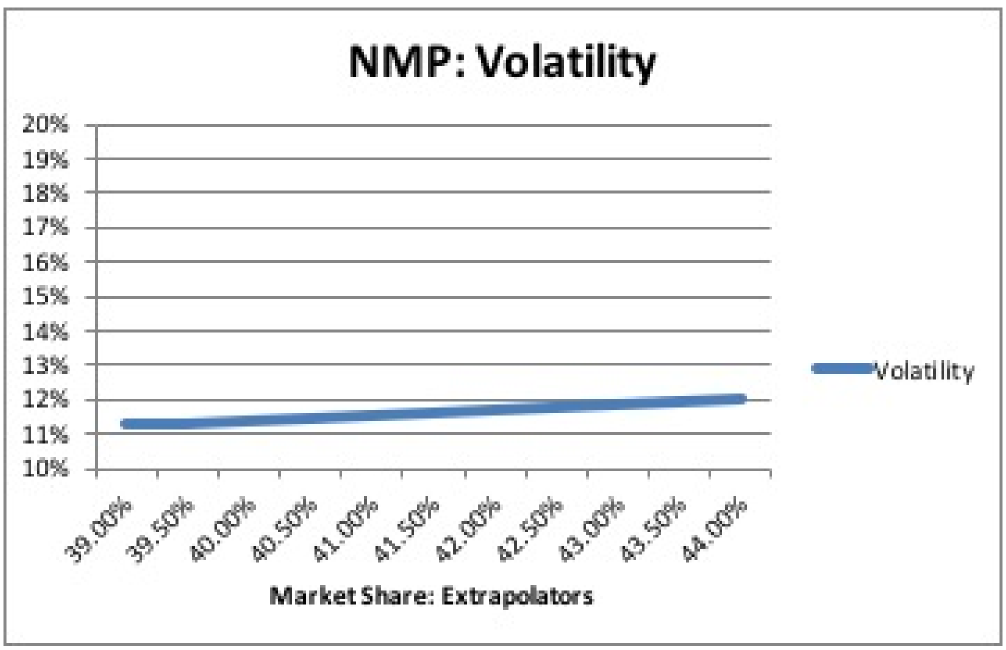


(b) No Market Power

Figure B.10: Extrapolators market share: excess return

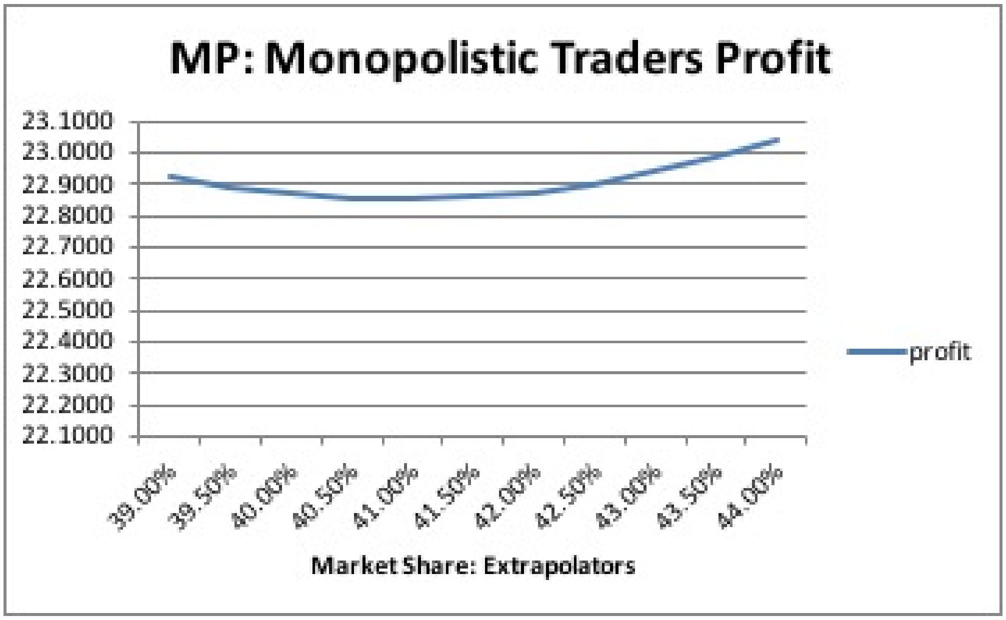


(a) Market Power

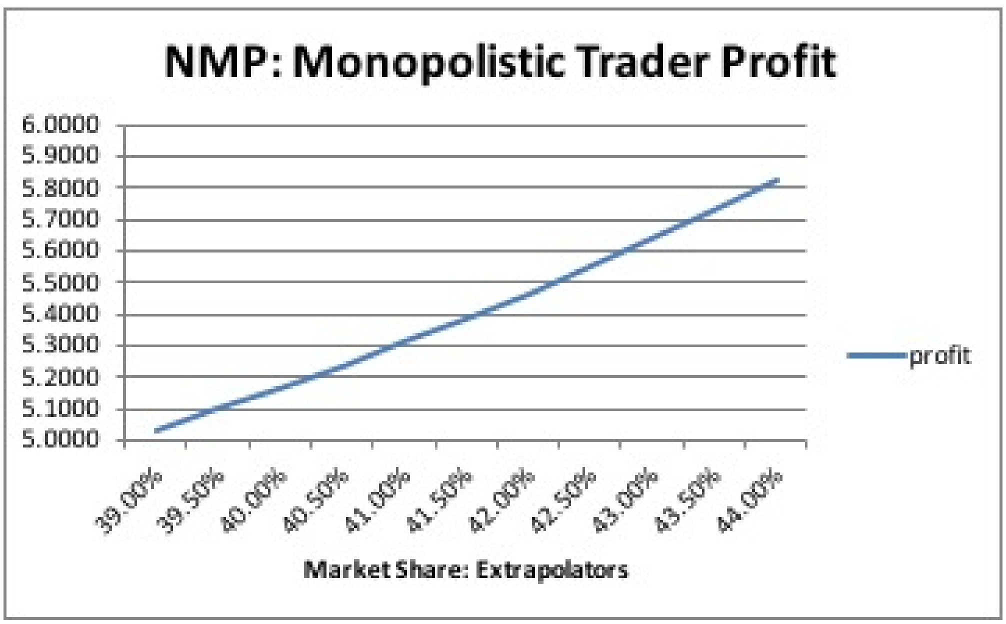


(b) No Market Power

Figure B.11: Extrapolators market share: volatility



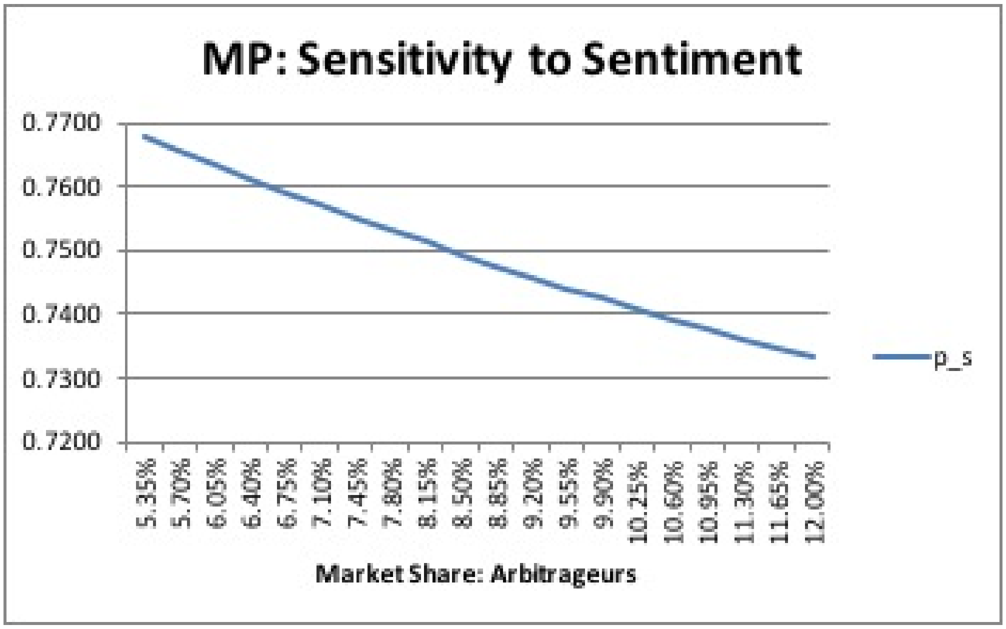
(a) Market Power



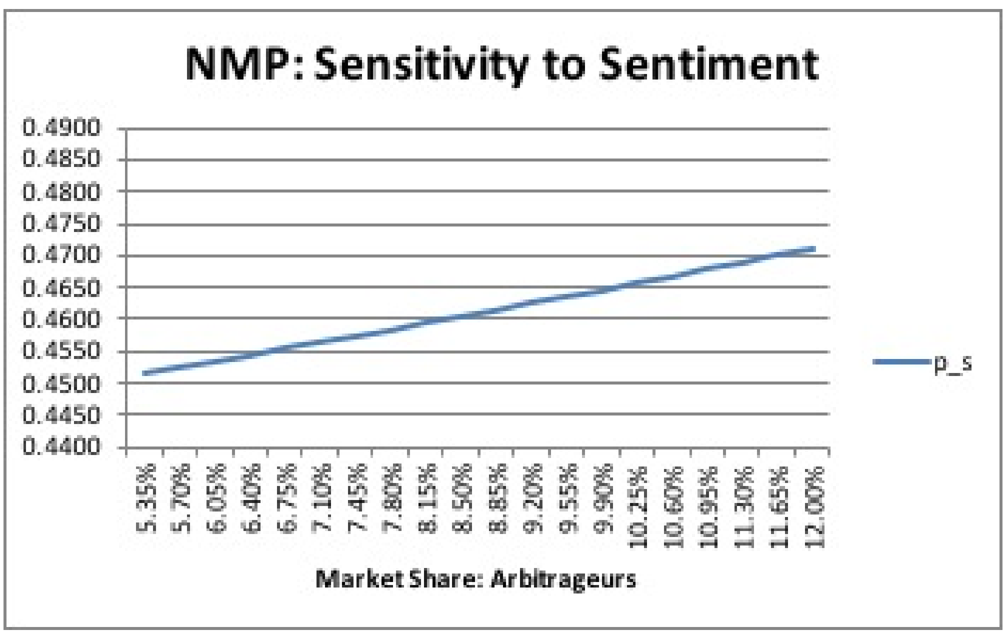
(b) No Market Power

Figure B.12: Extrapolators market share: profit

B.5 Arbitrageurs Market Share

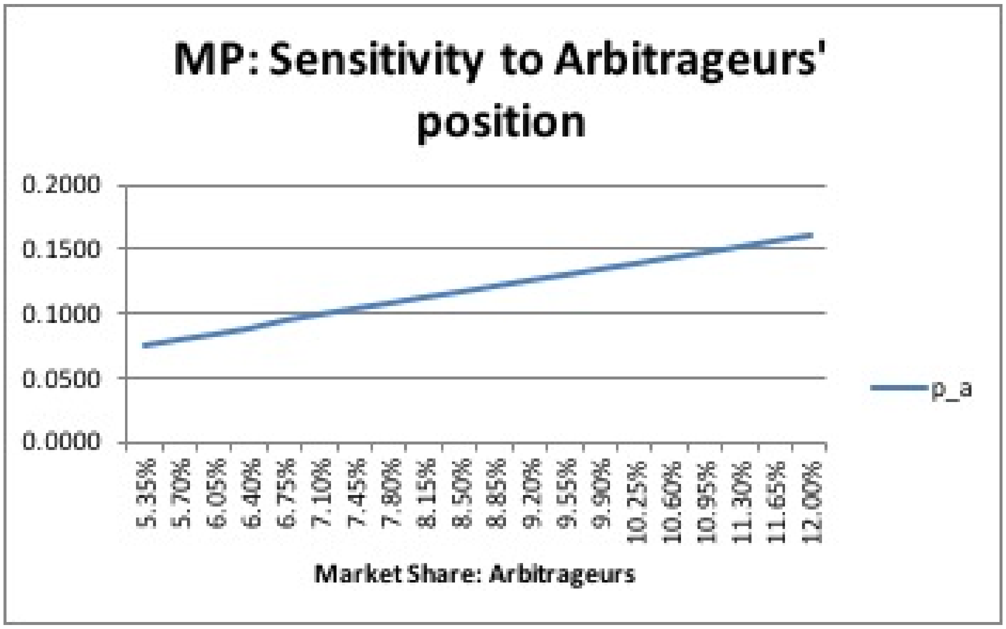


(a) Market Power

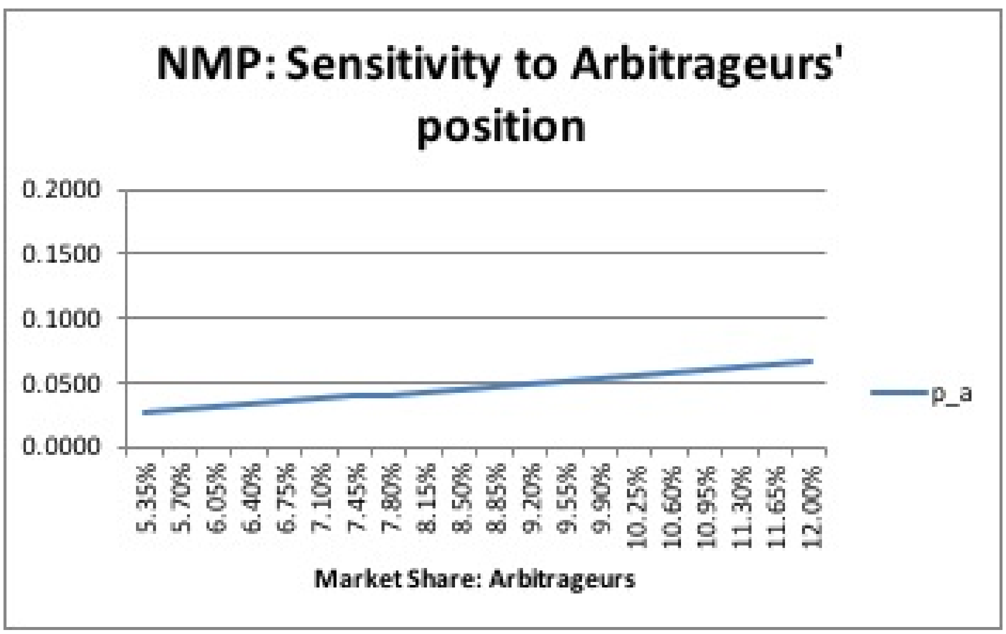


(b) No Market Power

Figure B.13: Arbitrageurs market share: sensitivity to sentiment

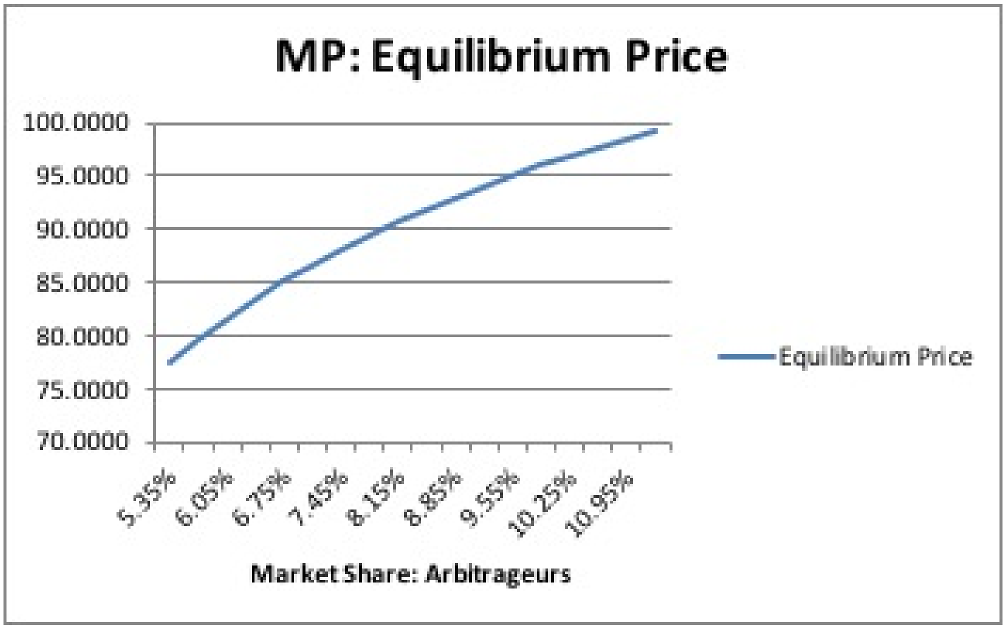


(a) Market Power

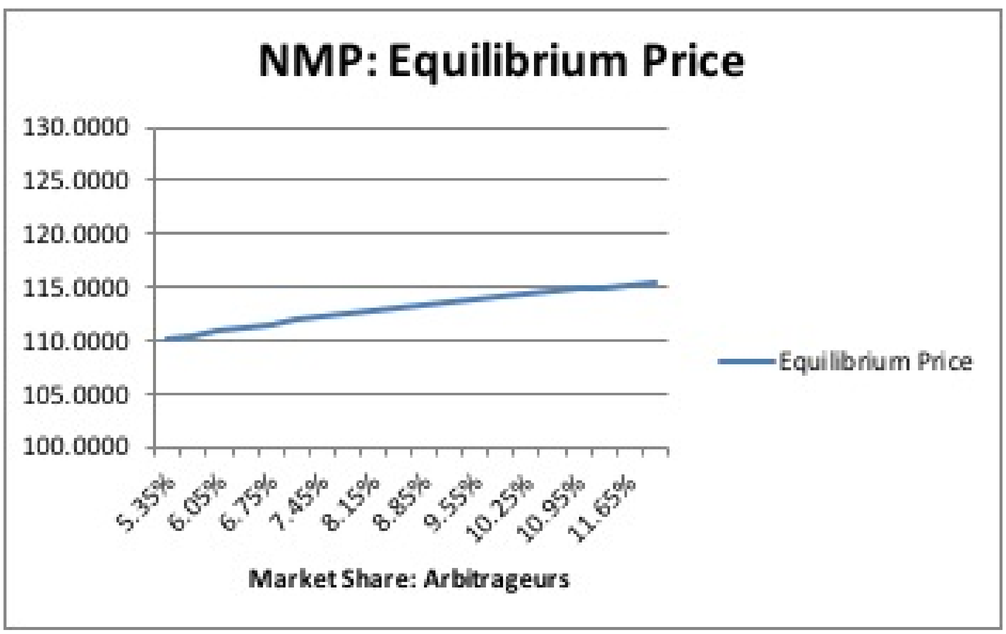


(b) No Market Power

Figure B.14: Arbitrageurs market share: sensitivity to arbitrageurs

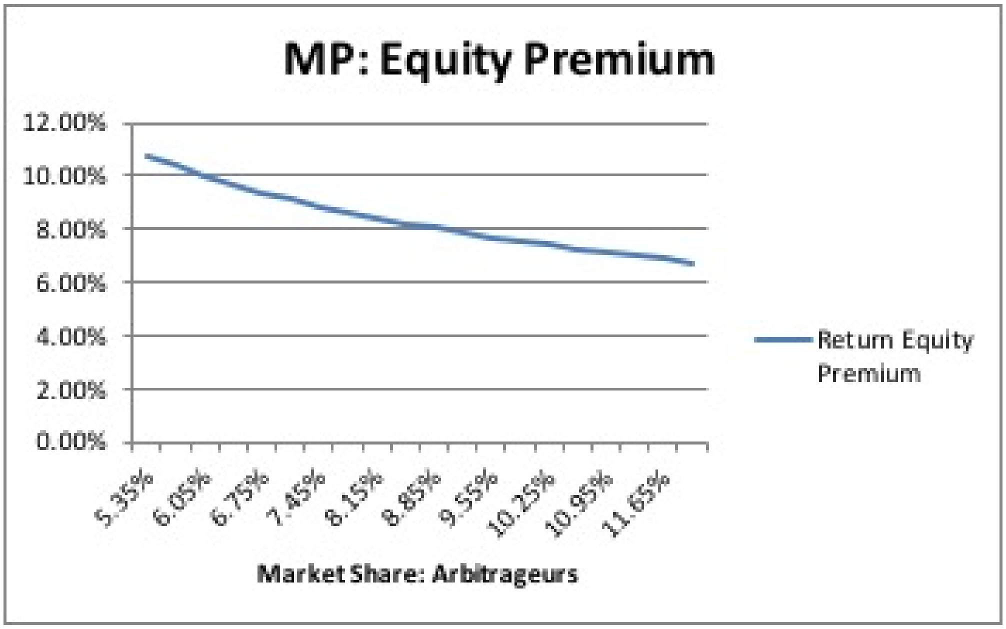


(a) Market Power

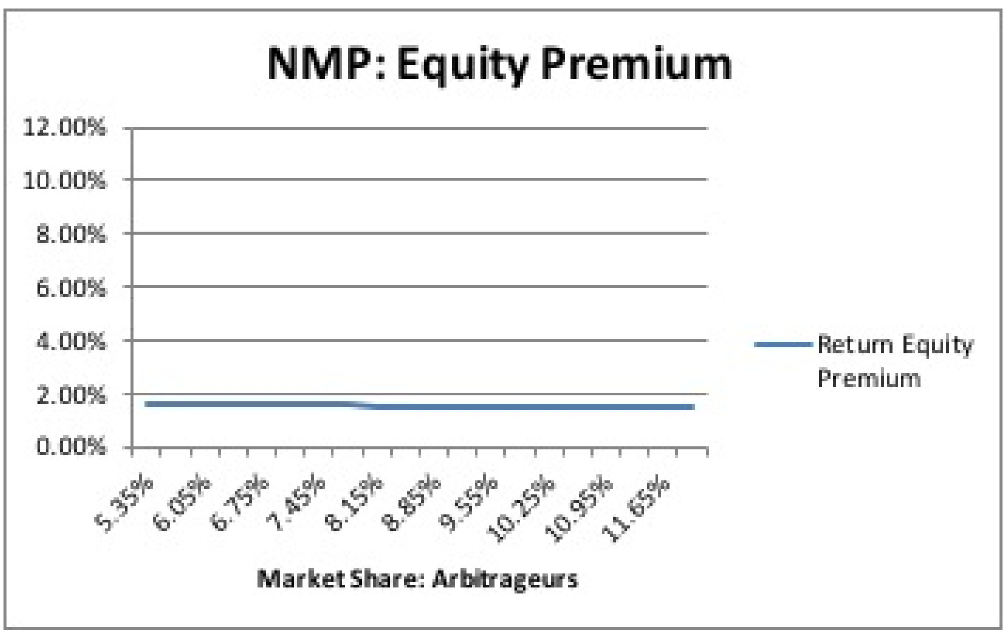


(b) No Market Power

Figure B.15: Arbitrageurs market share: equilibrium price

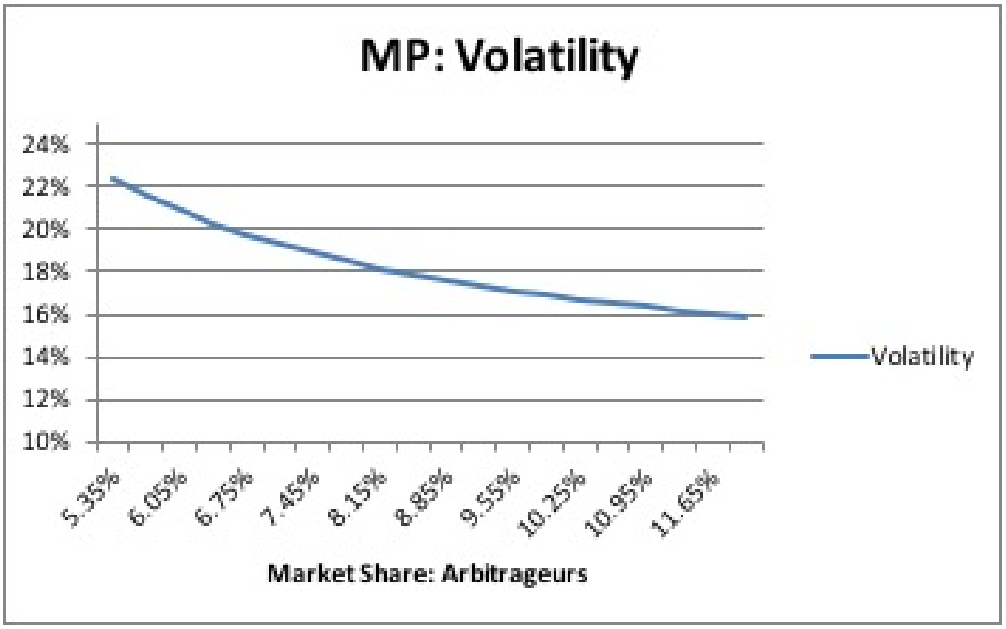


(a) Market Power

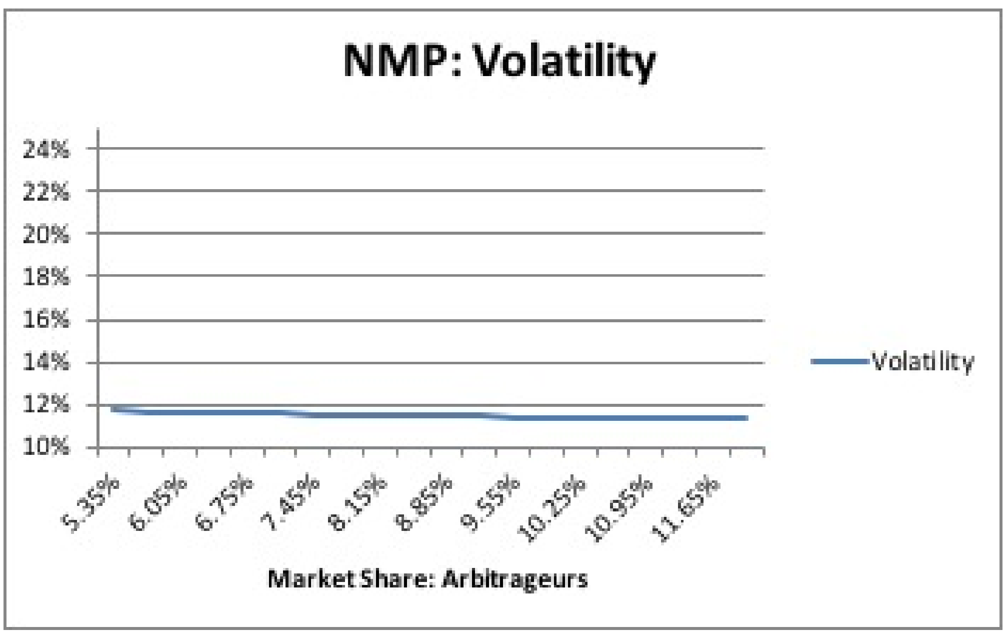


(b) No Market Power

Figure B.16: Arbitrageurs market share: excess return

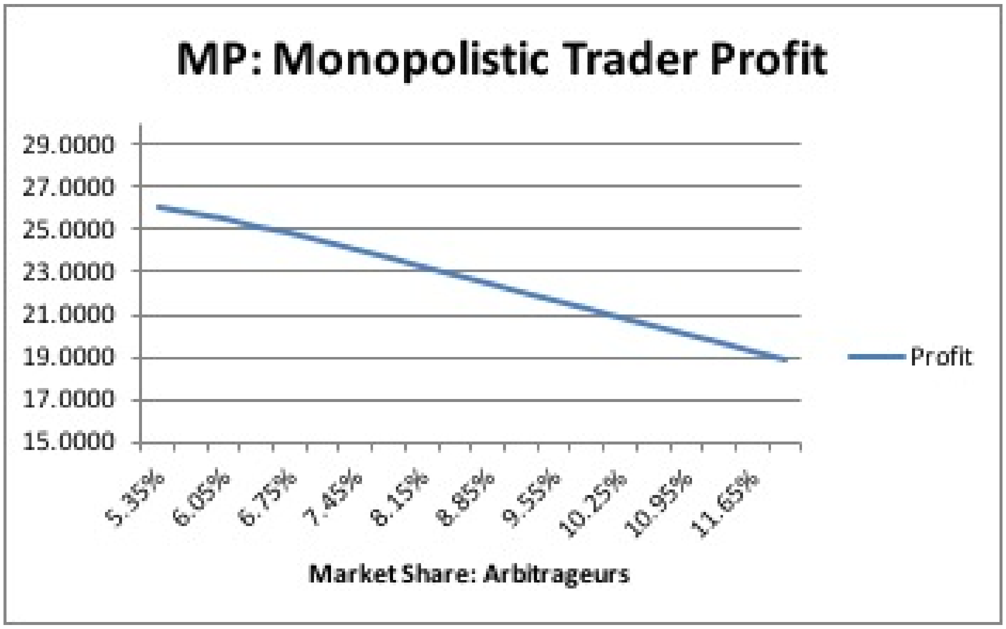


(a) Market Power

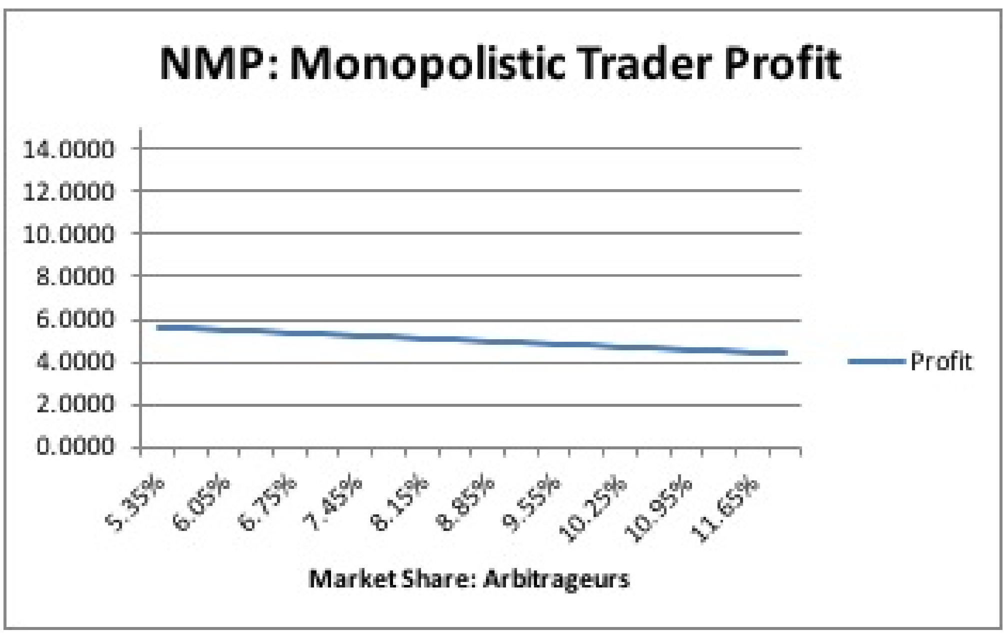


(b) No Market Power

Figure B.17: Arbitrageurs market share: volatility



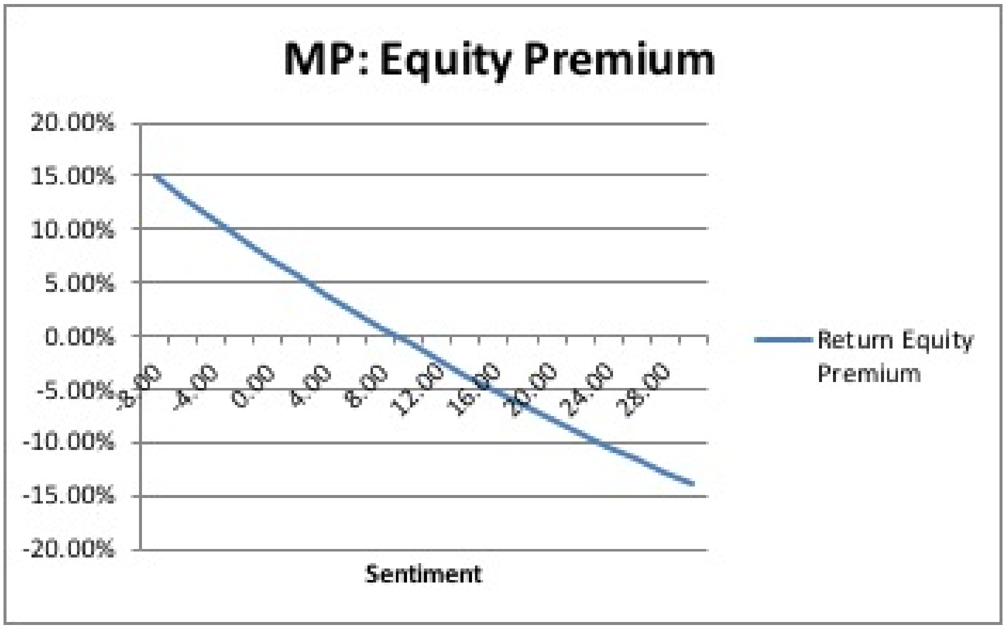
(a) Market Power



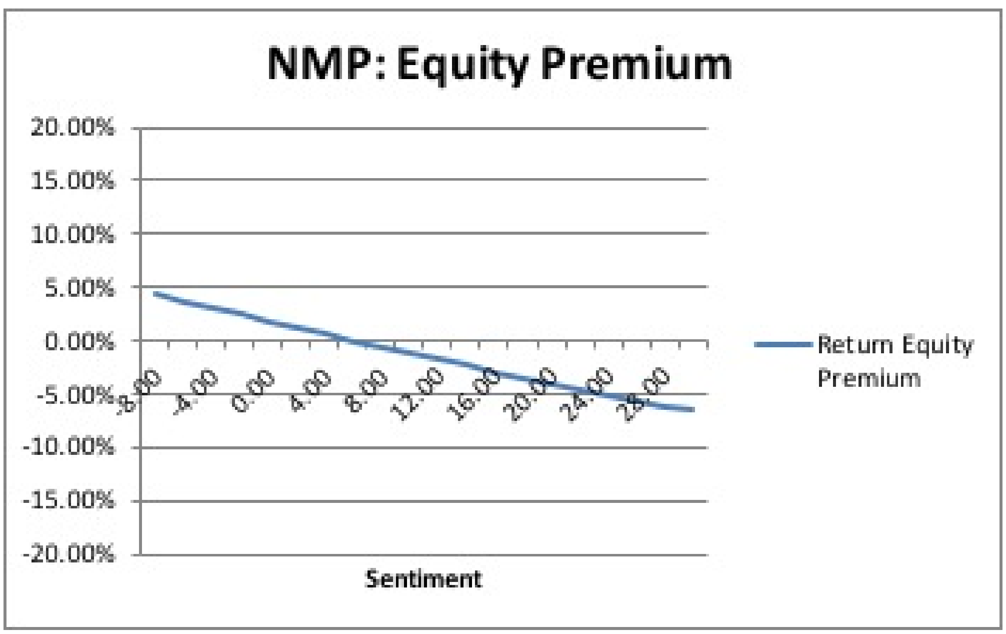
(b) No Market Power

Figure B.18: Arbitrageurs market share: profit

B.6 State Variables

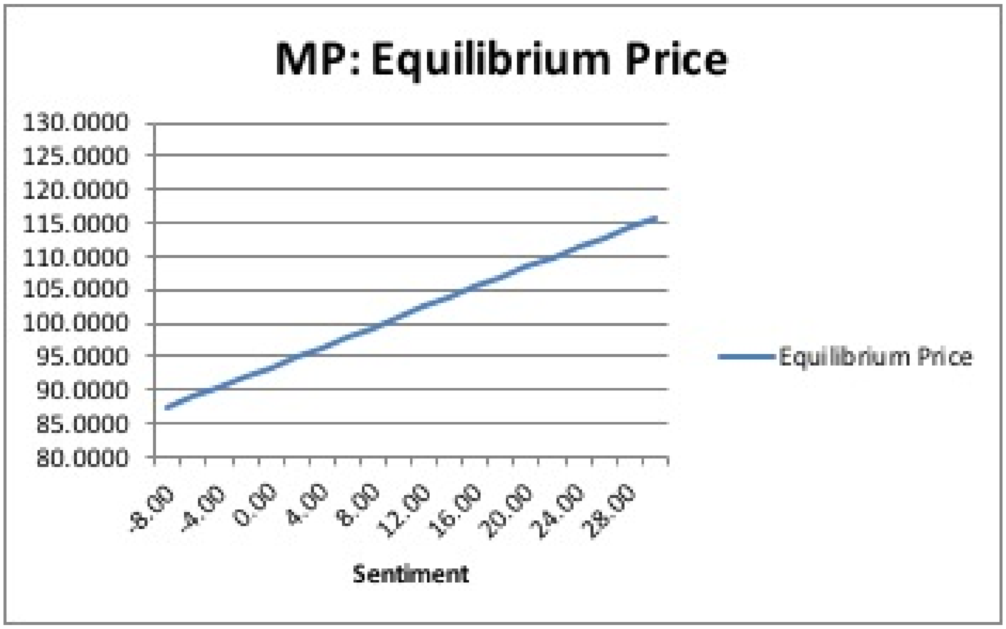


(a) Market Power

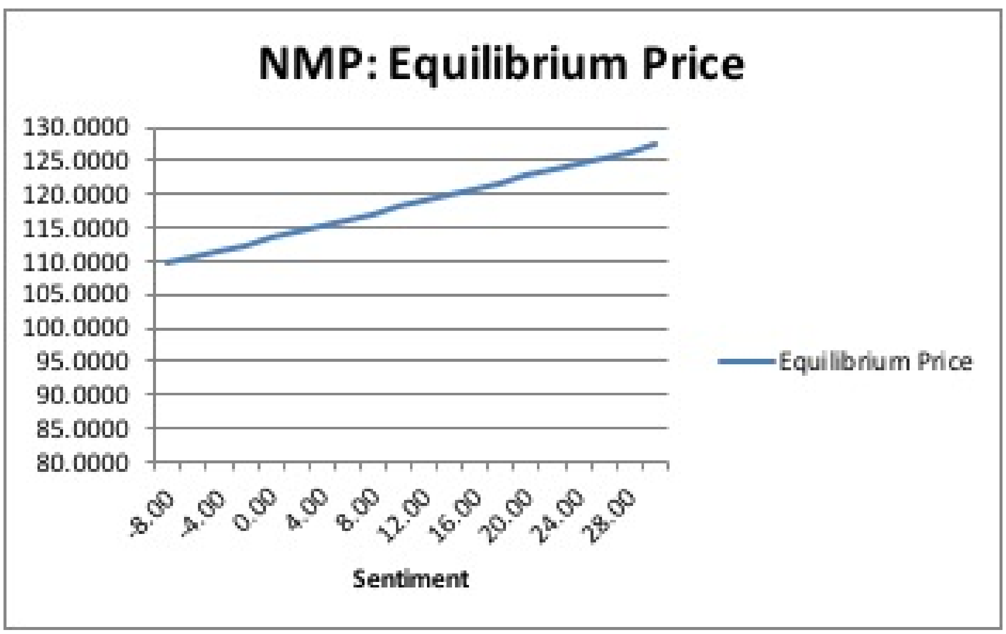


(b) No Market Power

Figure B.19: Sentiment and excess return

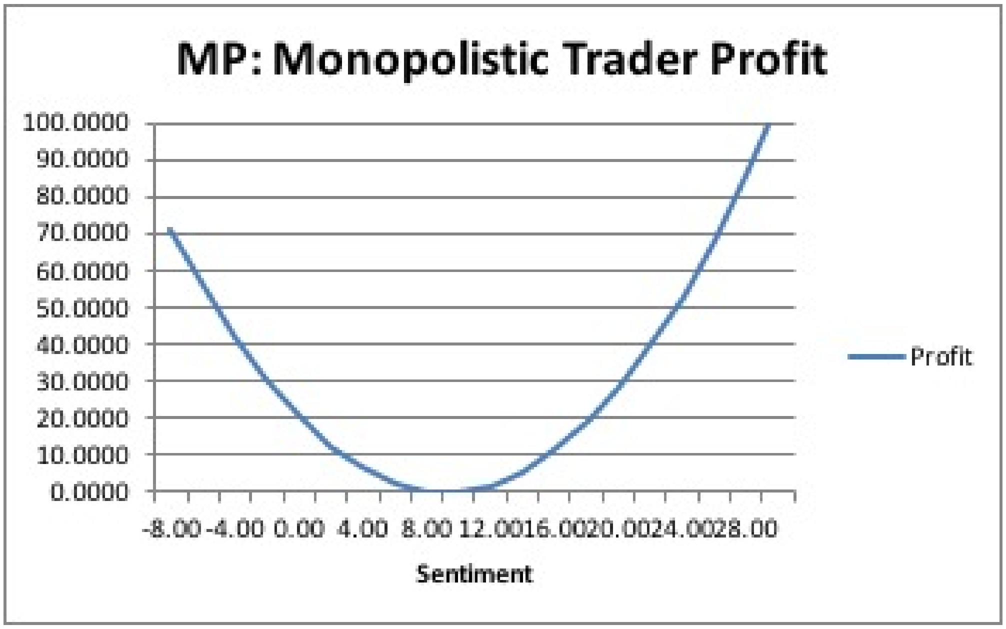


(a) Market Power

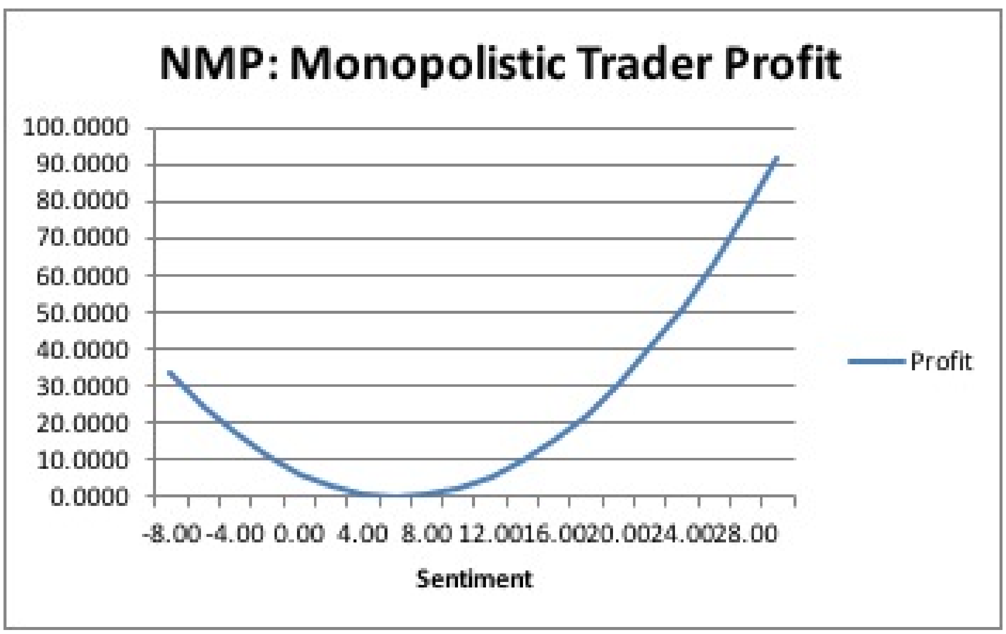


(b) No Market Power

Figure B.20: Sentiment and equilibrium price

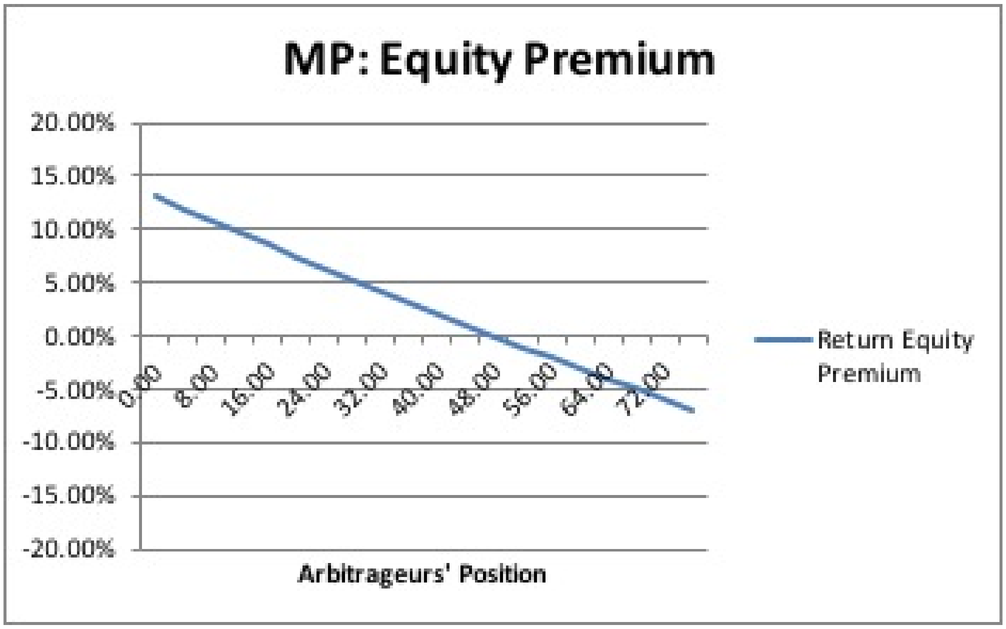


(a) Market Power

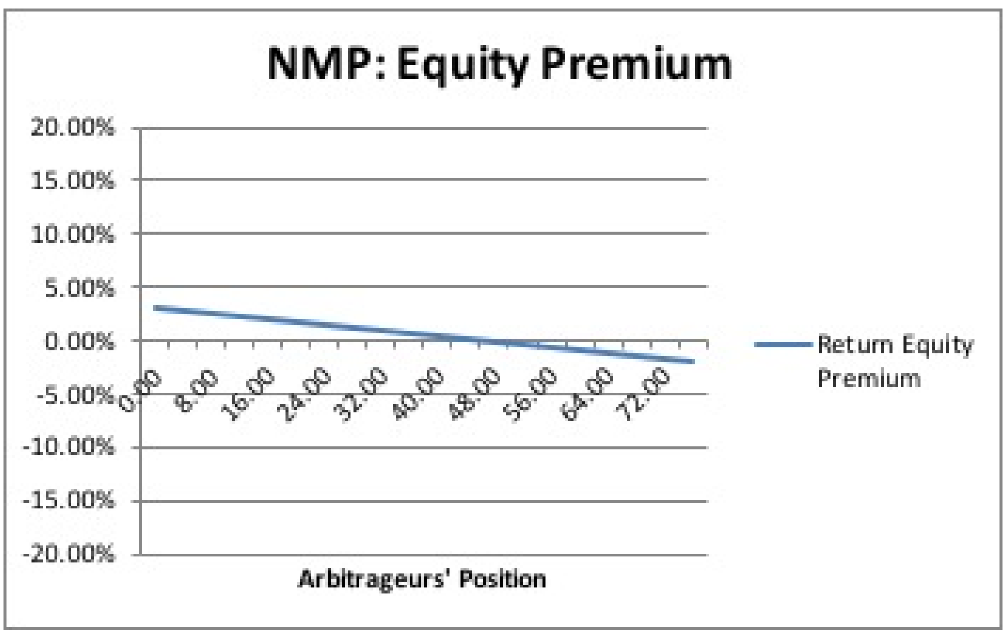


(b) No Market Power

Figure B.21: Sentiment and profit

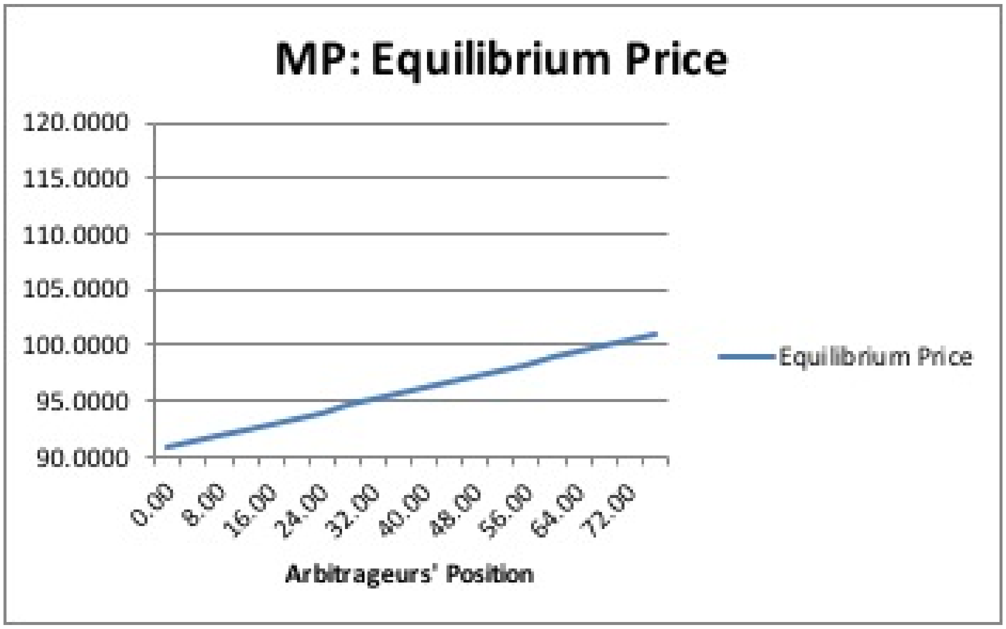


(a) Market Power

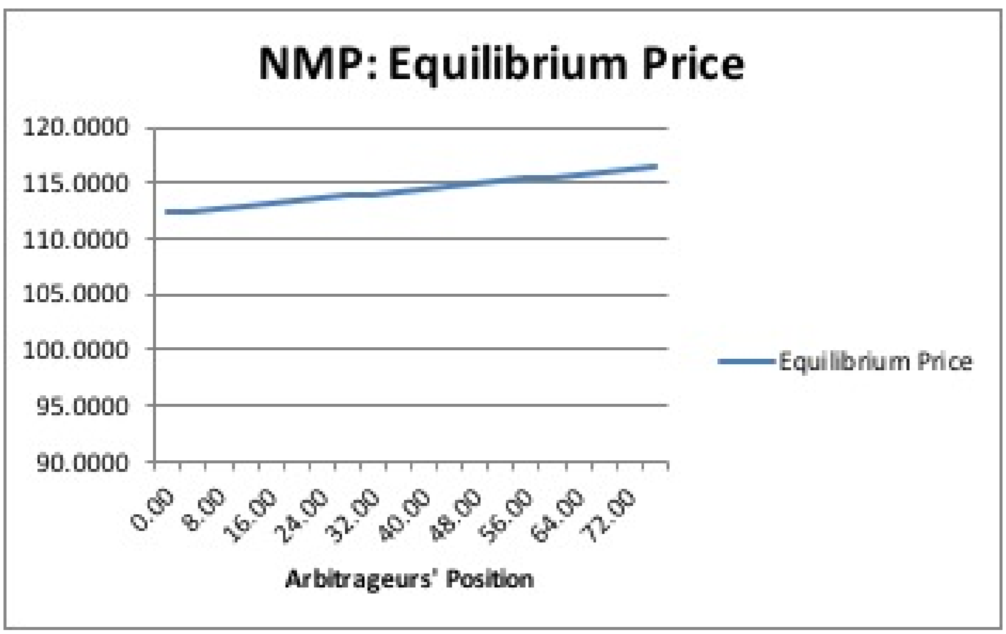


(b) No Market Power

Figure B.22: Arbitrageurs and excess return

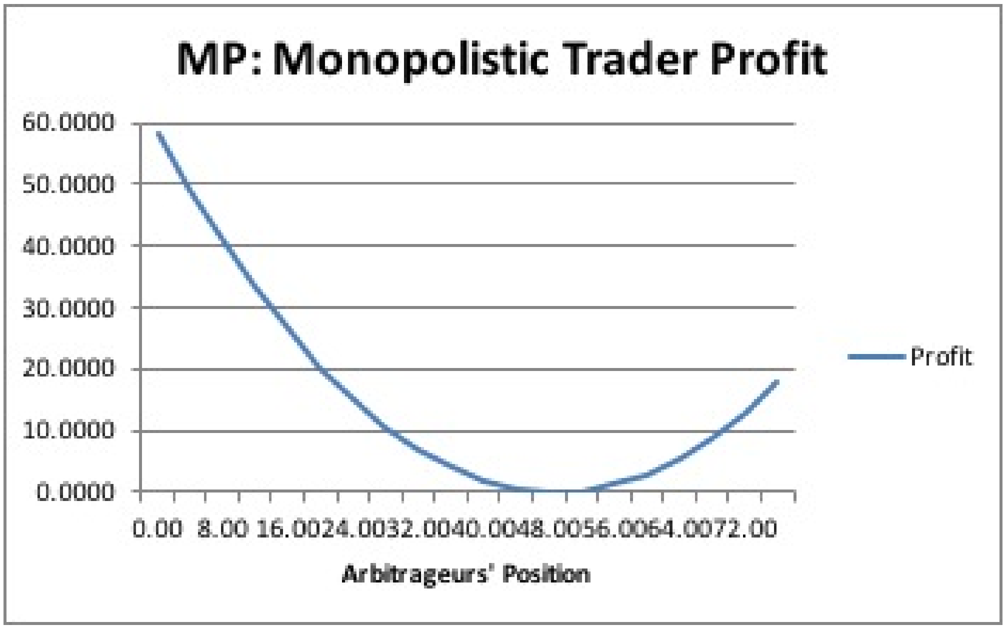


(a) Market Power

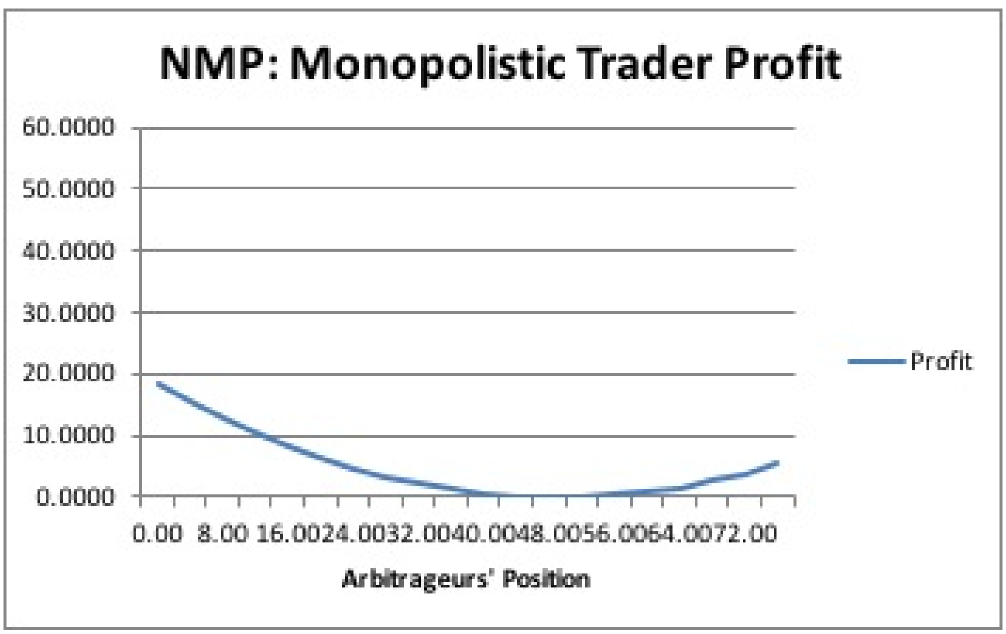


(b) No Market Power

Figure B.23: Arbitrageurs and equilibrium price



(a) Market Power



(b) No Market Power

Figure B.24: Arbitrageurs and profit

B.7 Matlab Code

B.7.1 Solving Non-Competitive MAX-CAPM

```
% Numerically Solve the equilibrium parameters
% The manipulator does have market power. They are aware that their
   demand
% will affect expected portfolio return.

% There are 15 parameters, 6 associate with extrapolators' optimal
% strategy, 6 associate with manipulator's optimal strategy, and the
   rest 3
% associate with the equilibrium price

% First we assign value to model parameters. Then we apply Matlab
   standard
% package to solve second order polynomial equations system with 15
% equations and 15 unknowns. Some condition may be applied to rule out
% multiple solutions.

% The standard packages we consider are: vpasolve

solutions = zeros(20,25);

% Parameter Value

% Extrapolators
```

```
beta= 0.5;
lambda0= 0.1;
lambda1= 0.9;

% Fundamental
r= 0.025;
g_D= 0.05; %normal
sigma_D= 0.25;
gamma= 0.1;
delta= 0.015;
Q= 5;

% Arbitrageurs
alpha= 1; %volatility of arbitrageur
sigma_a = 1;

P_r = -gamma*sigma_D^2*Q/r^2 + g_D/r^2;
% end of parameter values specification

for i = 1:20

% Market weight
mu0= 0.4;
mu1= 0.05+0.0035*i;
mu2= 1-mu0-mu1;

% Define variables
syms a_e b_e c_e d_e e_e f_e a_m b_m c_m d_m e_m f_m p_0 p_s p_a;
```

```
% define the system of polynomials to solve parameters of
   extrapolators'
% problem

e0e = lambda0 - r * p_0;
eSe = lambda1 - r * p_s;
eae = -r * p_a;

fact = (1-p_s*beta);
sigmap2 = (sigma_D^2/r^2+p_a^2*sigma_a^2)/fact^2;

f0e = (e0e+beta*d_e*sigmap2+e_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fSe =
    (eSe+2*a_e*beta*sigmap2+c_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fae =
    (eae+c_e*beta*sigmap2+2*b_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);

eqn1 = 0 == -r*a_e - r*gamma*fSe*eSe + 2*a_e*beta*(lambda1-1) +
   c_e*alpha + 1/2*(r*gamma)^2*fSe^2*sigmap2 + 2*a_e^2*beta^2*sigmap2
   + 1/2*c_e^2*sigma_a^2 ...
   - 2*a_e*fSe*r*gamma*beta*sigmap2 -
      c_e*fSe*r*gamma*p_a*sigma_a^2/fact +
      2*a_e*c_e*beta*p_a*sigma_a^2/fact;

eqn2 = 0 == -r*b_e - r*gamma*fae*eae - 2*b_e*alpha +
   1/2*(r*gamma)^2*fae^2*sigmap2 + 1/2*c_e^2*beta^2*sigmap2 +
```

```
    2*b_e^2*sigma_a^2 ...
    - c_e*fae*r*gamma*beta*sigmap2 -
       2*b_e*fae*r*gamma*p_a*sigma_a^2/fact +
       2*b_e*c_e*beta*p_a*sigma_a^2/fact;

eqn3 = 0 == -r*c_e - r*gamma*(fSe*eae+fae*eSe) + beta*c_e*(lambda1-1)
    + alpha*(2*b_e-c_e) + (r*gamma)^2*fSe*fae*sigmap2 +
    2*a_e*c_e*beta^2*sigmap2 ...
    + 2*b_e*c_e*sigma_a^2 - r*gamma*beta*sigmap2*(2*a_e*fae+c_e*fSe) -
       r*gamma*p_a*sigma_a^2/fact*(2*b_e*fSe+c_e*fae) ...
    + beta*p_a*sigma_a^2/fact*(4*a_e*b_e+c_e^2);

eqn4 = 0 == -r*d_e - r*gamma*(f0e*eSe+fSe*e0e) +
    beta*(2*a_e*lambda0+d_e*(lambda1-1)) + alpha*e_e +
    (r*gamma)^2*sigmap2*f0e*fSe + 2*a_e*d_e*beta^2*sigmap2 ...
    + c_e*e_e*sigma_a^2 - r*gamma*beta*sigmap2*(2*a_e*f0e+d_e*fSe) -
       r*gamma*p_a*sigma_a^2/fact*(c_e*f0e+e_e*fSe) +
       beta*p_a*sigma_a^2/fact*(2*a_e*e_e+c_e*d_e);

eqn5 = 0 == -r*e_e - r*gamma*(f0e*eae+fae*e0e) + beta*c_e*lambda0 -
    alpha*e_e + (r*gamma)^2*sigmap2*f0e*fae + c_e*d_e*beta^2*sigmap2 ...
    + 2*b_e*e_e*sigma_a^2 - r*gamma*beta*sigmap2*(c_e*f0e+d_e*fae) -
       r*gamma*p_a*sigma_a^2/fact*(2*b_e*f0e+e_e*fae) +
       beta*p_a*sigma_a^2/fact*(c_e*e_e+2*b_e*d_e);

eqn6 = 0 == r - delta - r*f_e - r*log(r*gamma) - r*gamma*f0e*e0e +
    beta*d_e*lambda0 + 1/2*(r*gamma)^2*f0e^2*sigmap2 +
    1/2*beta^2*sigmap2*(2*a_e+d_e^2) ...
```

```
    + 1/2*sigma_a^2*(2*b_e+e_e^2) - r*gamma*beta*d_e*f0e*sigmap2 -
        r*gamma*p_a*sigma_a^2/fact*e_e*f0e +
        beta*p_a*sigma_a^2/fact*(c_e+d_e*e_e);


% define the system of polynomials to solve parameters of manipulator's
% problem when they have market power

p0r = (g_D-gamma*Q*sigma_D^2)/r^2;

e0m = r*gamma*sigmap2*Q/mu0 -
    (lambda0+beta*d_e*sigmap2+e_e*p_a*sigma_a^2/fact) +
    (g_D/r+p_a*alpha*(p0r-p_0))/fact;
eSm = -(lambda1+2*a_e*beta*sigmap2+c_e*p_a*sigma_a^2/fact) -
    (p_s*beta+p_a*p_s*alpha)/fact;
eam = -mu1*r*gamma*sigmap2/mu0 -
    (c_e*beta*sigmap2+2*b_e*p_a*sigma_a^2/fact) -
    p_a*(p_a+1)*alpha/fact;
emm = -mu2*r*gamma*sigmap2/mu0;

p0m = beta/fact*(g_D+alpha*r*p_a*(p0r-p_0))/r;
pSm = -beta/fact*(1+p_a*p_s*alpha);
pam = -beta/fact*p_a*(p_a+1)*alpha;

f0m =
    (e0m+beta*d_m*sigmap2+e_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2-2*emm);
fSm =
    (eSm+2*a_m*beta*sigmap2+c_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2-2*emm);
```

```
fam =
   (eam+c_m*beta*sigmap2+2*b_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2-2*emm);

eqn7 = 0 == -r*a_m - r*gamma*fSm*eSm + 2*a_m*pSm - c_m*p_s*alpha +
   1/2*(r*gamma)^2*fSm^2*sigmap2 + 2*a_m^2*beta^2*sigmap2 +
   1/2*c_m^2*sigma_a^2 ...
   - 2*a_m*fSm*r*gamma*beta*sigmap2 -
      c_m*fSm*r*gamma*p_a*sigma_a^2/fact +
      2*a_m*c_m*beta*p_a*sigma_a^2/fact - r*gamma*emm*fSm^2;

eqn8 = 0 == -r*b_m - r*gamma*fam*eam + c_m*pam - 2*b_m*(p_a+1)*alpha +
   1/2*(r*gamma)^2*fam^2*sigmap2 + 1/2*c_m^2*beta^2*sigmap2 +
   2*b_m^2*sigma_a^2 ...
   - c_m*fam*r*gamma*beta*sigmap2 -
      2*b_m*fam*r*gamma*p_a*sigma_a^2/fact +
      2*b_m*c_m*beta*p_a*sigma_a^2/fact - r*gamma*emm*fam^2;

eqn9 = 0 == -r*c_m - r*gamma*(fSm*eam+fam*eSm) + (2*a_m*pam+c_m*pSm) -
   alpha*(2*b_m*p_s+c_m*(p_a+1)) + (r*gamma)^2*fSm*fam*sigmap2 +
   2*a_m*c_m*beta^2*sigmap2 ...
   + 2*b_m*c_m*sigma_a^2 - r*gamma*beta*sigmap2*(2*a_m*fam+c_m*fSm) -
      r*gamma*p_a*sigma_a^2/fact*(2*b_m*fSm+c_m*fam) ...
   + beta*p_a*sigma_a^2/fact*(4*a_m*b_m+c_m^2) - 2*r*gamma*emm*fSm*fam;

eqn10 = 0 == -r*d_m - r*gamma*(f0m*eSm+fSm*e0m) + (2*a_m*p0m+d_m*pSm)
   + alpha*(-e_m*p_s+(p0r-p_0)*c_m) + (r*gamma)^2*sigmap2*f0m*fSm +
   2*a_m*d_m*beta^2*sigmap2 + c_m*e_m*sigma_a^2 ...
```

```
    - r*gamma*beta*sigmap2*(2*a_m*f0m+d_m*fSm) -
      r*gamma*p_a*sigma_a^2/fact*(c_m*f0m+e_m*fSm) +
      beta*p_a*sigma_a^2/fact*(2*a_m*e_m+c_m*d_m) ...
    - 2*r*gamma*emm*f0m*fSm;

eqn11 = 0 == -r*e_m - r*gamma*(f0m*eam+fam*e0m) + (c_m*p0m+d_m*pam) +
    alpha*(2*b_m*(p0r-p_0)-e_m*(p_a+1)) + (r*gamma)^2*sigmap2*f0m*fam +
    c_m*d_m*beta^2*sigmap2 + 2*b_m*e_m*sigma_a^2 ...
    - r*gamma*beta*sigmap2*(c_m*f0m+d_m*fam) -
      r*gamma*p_a*sigma_a^2/fact*(2*b_m*f0m+e_m*fam) +
      beta*p_a*sigma_a^2/fact*(c_m*e_m+2*b_m*d_m) ...
    - 2*r*gamma*emm*f0m*fam;

eqn12 = 0 == r - delta - r*f_m - r*log(r*gamma) - r*gamma*f0m*e0m +
    d_m*p0m + alpha*e_m*(p0r-p_0) + 1/2*(r*gamma)^2*f0m^2*sigmap2 +
    1/2*beta^2*sigmap2*(2*a_m+d_m^2) ...
    + 1/2*sigma_a^2*(2*b_m+e_m^2) - r*gamma*beta*d_m*f0m*sigmap2 -
      r*gamma*p_a*sigma_a^2/fact*e_m*f0m +
      beta*p_a*sigma_a^2/fact*(c_m+d_m*e_m) ...
    - r*gamma*emm*f0m^2;


% market clearing condition

eqn13 = 0 == mu0*f0e + mu2*f0m - Q;

eqn14 = 0 == mu0*fSe + mu2*fSm;
```

```
eqn15 = 0 == mu0*fae + mu1 + mu2*fam;

% numerical polynomial solver

S =
   vpasolve([eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,eqn7,eqn8,eqn9,eqn10,eqn11,eqn12,eqn13,e
   [a_e,b_e,c_e,d_e,e_e,f_e,a_m,b_m,c_m,d_m,e_m,f_m,p_0,p_s,p_a]);


a_e= S.a_e;
b_e= S.b_e;
c_e= S.c_e;
d_e= S.d_e;
e_e= S.e_e;
f_e= S.f_e;
a_m= S.a_m;
b_m= S.b_m;
c_m= S.c_m;
d_m= S.d_m;
e_m= S.e_m;
f_m= S.f_m;
p_0= S.p_0;
p_s= S.p_s;
p_a= S.p_a;

e0e = lambda0 - r * p_0;
eSe = lambda1 - r * p_s;
eae = -r * p_a;
```

```
fact = (1-p_s*beta);
sigmap2 = (sigma_D^2/r^2+p_a^2*sigma_a^2)/fact^2;

f0e = (e0e+beta*d_e*sigmap2+e_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fSe =
   (eSe+2*a_e*beta*sigmap2+c_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fae =
   (eae+c_e*beta*sigmap2+2*b_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);

p0r = (g_D-gamma*Q*sigma_D^2)/r^2;

e0m = r*gamma*sigmap2*Q/mu0 -
   (lambda0+beta*d_e*sigmap2+e_e*p_a*sigma_a^2/fact) +
   (g_D/r+p_a*alpha*(p0r-p_0))/fact;
eSm = -(lambda1+2*a_e*beta*sigmap2+c_e*p_a*sigma_a^2/fact) -
   (p_s*beta+p_a*p_s*alpha)/fact;
eam = -mu1*r*gamma*sigmap2/mu0 -
   (c_e*beta*sigmap2+2*b_e*p_a*sigma_a^2/fact) -
   p_a*(p_a+1)*alpha/fact;
emm = -mu2*r*gamma*sigmap2/mu0;

p0m = beta/fact*(g_D+alpha*r*p_a*(p0r-p_0))/r;
pSm = -beta/fact*(1+p_a*p_s*alpha);
pam = -beta/fact*p_a*(p_a+1)*alpha;

f0m =
   (e0m+beta*d_m*sigmap2+e_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2-2*emm);
```

```
fSm =
    (eSm+2*a_m*beta*sigmap2+c_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2-2*emm);
fam =
    (eam+c_m*beta*sigmap2+2*b_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2-2*emm);


solutions(i,1)= a_e;
solutions(i,2)= b_e;
solutions(i,3)= c_e;
solutions(i,4)= d_e;
solutions(i,5)= e_e;
solutions(i,6)= f_e;
solutions(i,7)= a_m;
solutions(i,8)= b_m;
solutions(i,9)= c_m;
solutions(i,10)= d_m;
solutions(i,11)= e_m;
solutions(i,12)= f_m;
solutions(i,13)= p_0;
solutions(i,14)= p_s;
solutions(i,15)= p_a;
solutions(i,16)= f0e;
solutions(i,17)= fSe;
solutions(i,18)= fae;
solutions(i,19)= f0m;
solutions(i,20)= fSm;
solutions(i,21)= fam;
solutions(i,22)= e0m;
```

```
solutions(i,23)= eSm;
solutions(i,24)= eam;
solutions(i,25)= emm;

end

xlswrite('MP_Arbitraguer_Wt.xlsx',solutions);
```

B.7.2 Solving Competitive MAX-CAPM

```
% Numerically solve the equilibrium parameters
% The manipulator doesn’t have market power.

% There are 15 parameters, 6 associate with extrapolators’ optimal
% strategy, 6 associate with manipulator’s optimal strategy, and the
   rest 3
% associate with the equilibrium price

% First we assign value to model parameters. Then we apply Matlab
   standard
% package to solve second order polynomial equations system with 15
% equations and 15 unknowns. Some condition may be applied to rule out
% multiple solutions.

% The standard packages we consider are: vpasolve.

solutions = zeros(20,24);

% Parameter Value

% Extrapolators
beta= 0.5;
lambda0= 0.1;
lambda1= 0.9;

% Fundamental
```

```
r= 0.025;
g_D= 0.05; %normal
sigma_D= 0.25;
gamma= 0.1;
delta= 0.015;
Q= 5;

% Arbitrageurs
alpha= 1; %volatility of arbitrageur
sigma_a = 1;

% end of parameter values specification

for i = 1:20

% Market weight
mu0= 0.4;
mu1= 0.05+0.0035*i;
mu2= 1-mu0-mu1;

% Define variables
syms a_e b_e c_e d_e e_e f_e a_m b_m c_m d_m e_m f_m p_0 p_s p_a;

% define the system of polynomials to solve parameters of
   extrapolators'
% problem

e0e = lambda0 - r * p_0;
```

```
eSe = lambda1 - r * p_s;
eae = -r * p_a;

fact = (1-p_s*beta);
sigmap2 = (sigma_D^2/r^2+p_a^2*sigma_a^2)/fact^2;

f0e = (e0e+beta*d_e*sigmap2+e_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fSe =
   (eSe+2*a_e*beta*sigmap2+c_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fae =
   (eae+c_e*beta*sigmap2+2*b_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);

eqn1 = 0 == -r*a_e - r*gamma*fSe*eSe + 2*a_e*beta*(lambda1-1) +
   c_e*alpha + 1/2*(r*gamma)^2*fSe^2*sigmap2 + 2*a_e^2*beta^2*sigmap2
   + 1/2*c_e^2*sigma_a^2 ...
   - 2*a_e*fSe*r*gamma*beta*sigmap2 -
      c_e*fSe*r*gamma*p_a*sigma_a^2/fact +
      2*a_e*c_e*beta*p_a*sigma_a^2/fact;

eqn2 = 0 == -r*b_e - r*gamma*fae*eae - 2*b_e*alpha +
   1/2*(r*gamma)^2*fae^2*sigmap2 + 1/2*c_e^2*beta^2*sigmap2 +
   2*b_e^2*sigma_a^2 ...
   - c_e*fae*r*gamma*beta*sigmap2 -
      2*b_e*fae*r*gamma*p_a*sigma_a^2/fact +
      2*b_e*c_e*beta*p_a*sigma_a^2/fact;

eqn3 = 0 == -r*c_e - r*gamma*(fSe*eae+fae*eSe) + beta*c_e*(lambda1-1)
   + alpha*(2*b_e-c_e) + (r*gamma)^2*fSe*fae*sigmap2 +
```

```
   2*a_e*c_e*beta^2*sigmap2 ...
   + 2*b_e*c_e*sigma_a^2 - r*gamma*beta*sigmap2*(2*a_e*fae+c_e*fSe) -
      r*gamma*p_a*sigma_a^2/fact*(2*b_e*fSe+c_e*fae) ...
   + beta*p_a*sigma_a^2/fact*(4*a_e*b_e+c_e^2);

eqn4 = 0 == -r*d_e - r*gamma*(f0e*eSe+fSe*e0e) +
   beta*(2*a_e*lambda0+d_e*(lambda1-1)) + alpha*e_e +
   (r*gamma)^2*sigmap2*f0e*fSe + 2*a_e*d_e*beta^2*sigmap2 ...
   + c_e*e_e*sigma_a^2 - r*gamma*beta*sigmap2*(2*a_e*f0e+d_e*fSe) -
      r*gamma*p_a*sigma_a^2/fact*(c_e*f0e+e_e*fSe) +
      beta*p_a*sigma_a^2/fact*(2*a_e*e_e+c_e*d_e);

eqn5 = 0 == -r*e_e - r*gamma*(f0e*eae+fae*e0e) + beta*c_e*lambda0 -
   alpha*e_e + (r*gamma)^2*sigmap2*f0e*fae + c_e*d_e*beta^2*sigmap2 ...
   + 2*b_e*e_e*sigma_a^2 - r*gamma*beta*sigmap2*(c_e*f0e+d_e*fae) -
      r*gamma*p_a*sigma_a^2/fact*(2*b_e*f0e+e_e*fae) +
      beta*p_a*sigma_a^2/fact*(c_e*e_e+2*b_e*d_e);

eqn6 = 0 == r - delta - r*f_e - r*log(r*gamma) - r*gamma*f0e*e0e +
   beta*d_e*lambda0 + 1/2*(r*gamma)^2*f0e^2*sigmap2 +
   1/2*beta^2*sigmap2*(2*a_e+d_e^2) ...
   + 1/2*sigma_a^2*(2*b_e+e_e^2) - r*gamma*beta*d_e*f0e*sigmap2 -
      r*gamma*p_a*sigma_a^2/fact*e_e*f0e +
      beta*p_a*sigma_a^2/fact*(c_e+d_e*e_e);

% define the system of polynomials to solve parameters of manipulator's
% problem
```

```
p0r = (g_D-gamma*Q*sigma_D^2)/r^2;


e0m = -r*p_0 + (g_D/r+alpha*p_a*(p0r-p_0))/fact;

eSm = -r*p_s - (p_s*beta+p_a*p_s*alpha)/fact;

eam = -r*p_a - p_a*(p_a+1)*alpha/fact;


p0m = beta/fact*(g_D+alpha*r*p_a*(p0r-p_0))/r;

pSm = -beta/fact*(1+p_a*p_s*alpha);

pam = -beta/fact*p_a*(p_a+1)*alpha;


f0m = (e0m+beta*d_m*sigmap2+e_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);

fSm =
    (eSm+2*a_m*beta*sigmap2+c_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);

fam =
    (eam+c_m*beta*sigmap2+2*b_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);


eqn7 = 0 == -r*a_m - r*gamma*fSm*eSm + 2*a_m*pSm - c_m*p_s*alpha +
    1/2*(r*gamma)^2*fSm^2*sigmap2 + 2*a_m^2*beta^2*sigmap2 +
    1/2*c_m^2*sigma_a^2 ...
    - 2*a_m*fSm*r*gamma*beta*sigmap2 -
        c_m*fSm*r*gamma*p_a*sigma_a^2/fact +
        2*a_m*c_m*beta*p_a*sigma_a^2/fact;


eqn8 = 0 == -r*b_m - r*gamma*fam*eam + c_m*pam - 2*b_m*alpha*(p_a+1) +
    1/2*(r*gamma)^2*fam^2*sigmap2 + 1/2*c_m^2*beta^2*sigmap2 +
    2*b_m^2*sigma_a^2 ...
    - c_m*fam*r*gamma*beta*sigmap2 -
        2*b_m*fam*r*gamma*p_a*sigma_a^2/fact +
```

```
        2*b_m*c_m*beta*p_a*sigma_a^2/fact;


eqn9 = 0 == -r*c_m - r*gamma*(fSm*eam+fam*eSm) + (2*a_m*pam+c_m*pSm) -
    alpha*(2*b_m*p_s+c_m*(p_a+1)) + (r*gamma)^2*fSm*fam*sigmap2 +
    2*a_m*c_m*beta^2*sigmap2 ...
    + 2*b_m*c_m*sigma_a^2 - r*gamma*beta*sigmap2*(2*a_m*fam+c_m*fSm) -
        r*gamma*p_a*sigma_a^2/fact*(2*b_m*fSm+c_m*fam) +
        beta*p_a*sigma_a^2/fact*(4*a_m*b_m+c_m^2);


eqn10 = 0 == -r*d_m - r*gamma*(f0m*eSm+fSm*e0m) + (2*a_m*p0m+d_m*pSm)
    + alpha*(-e_m*p_s+(p0r-p_0)*c_m) + (r*gamma)^2*sigmap2*f0m*fSm +
    2*a_m*d_m*beta^2*sigmap2 + c_m*e_m*sigma_a^2 ...
    - r*gamma*beta*sigmap2*(2*a_m*f0m+d_m*fSm) -
        r*gamma*p_a*sigma_a^2/fact*(c_m*f0m+e_m*fSm) +
        beta*p_a*sigma_a^2/fact*(2*a_m*e_m+c_m*d_m);


eqn11 = 0 == -r*e_m - r*gamma*(f0m*eam+fam*e0m) + (c_m*p0m+d_m*pam) +
    alpha*(2*b_m*(p0r-p_0)-e_m*(p_a+1)) + (r*gamma)^2*sigmap2*f0m*fam +
    c_m*d_m*beta^2*sigmap2 + 2*b_m*e_m*sigma_a^2 ...
    - r*gamma*beta*sigmap2*(c_m*f0m+d_m*fam) -
        r*gamma*p_a*sigma_a^2/fact*(2*b_m*f0m+e_m*fam) +
        beta*p_a*sigma_a^2/fact*(c_m*e_m+2*b_m*d_m);


eqn12 = 0 == r - delta - r*f_m - r*log(r*gamma) - r*gamma*f0m*e0m +
    d_m*p0m + alpha*e_m*(p0r-p_0) + 1/2*(r*gamma)^2*f0m^2*sigmap2 +
    1/2*beta^2*sigmap2*(2*a_m+d_m^2) ...
    + 1/2*sigma_a^2*(2*b_m+e_m^2) - r*gamma*beta*d_m*f0m*sigmap2 -
        r*gamma*p_a*sigma_a^2/fact*e_m*f0m +
```

```
      beta*p_a*sigma_a^2/fact*(c_m+d_m*e_m);

% market clearing condition

eqn13 = 0 == mu0*f0e + mu2*f0m - Q;

eqn14 = 0 == mu0*fSe + mu2*fSm;

eqn15 = 0 == mu0*fae + mu1 + mu2*fam;

% numerical polynomial solver

S =
    vpasolve([eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,eqn7,eqn8,eqn9,eqn10,eqn11,eqn12,eqn13,e
    [a_e,b_e,c_e,d_e,e_e,f_e,a_m,b_m,c_m,d_m,e_m,f_m,p_0,p_s,p_a]);

a_e= S.a_e;
b_e= S.b_e;
c_e= S.c_e;
d_e= S.d_e;
e_e= S.e_e;
f_e= S.f_e;
a_m= S.a_m;
b_m= S.b_m;
c_m= S.c_m;
d_m= S.d_m;
e_m= S.e_m;
f_m= S.f_m;
```

```
p_0= S.p_0;
p_s= S.p_s;
p_a= S.p_a;

e0e = lambda0 - r * p_0;
eSe = lambda1 - r * p_s;
eae = -r * p_a;

fact = (1-p_s*beta);
sigmap2 = (sigma_D^2/r^2+p_a^2*sigma_a^2)/fact^2;

f0e = (e0e+beta*d_e*sigmap2+e_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fSe =
   (eSe+2*a_e*beta*sigmap2+c_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fae =
   (eae+c_e*beta*sigmap2+2*b_e*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);

p0r = (g_D-gamma*Q*sigma_D^2)/r^2;

e0m = -r*p_0 + (g_D/r+alpha*p_a*(p0r-p_0))/fact;
eSm = -r*p_s - (p_s*beta+p_a*p_s*alpha)/fact;
eam = -r*p_a - p_a*(p_a+1)*alpha/fact;

p0m = beta/fact*(g_D+alpha*r*p_a*(p0r-p_0))/r;
pSm = -beta/fact*(1+p_a*p_s*alpha);
pam = -beta/fact*p_a*(p_a+1)*alpha;

f0m = (e0m+beta*d_m*sigmap2+e_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
```

```
fSm =
    (eSm+2*a_m*beta*sigmap2+c_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);
fam =
    (eam+c_m*beta*sigmap2+2*b_m*p_a*sigma_a^2/fact)/(r*gamma*sigmap2);

solutions(i,1)= a_e;
solutions(i,2)= b_e;
solutions(i,3)= c_e;
solutions(i,4)= d_e;
solutions(i,5)= e_e;
solutions(i,6)= f_e;
solutions(i,7)= a_m;
solutions(i,8)= b_m;
solutions(i,9)= c_m;
solutions(i,10)= d_m;
solutions(i,11)= e_m;
solutions(i,12)= f_m;
solutions(i,13)= p_0;
solutions(i,14)= p_s;
solutions(i,15)= p_a;
solutions(i,16)= f0e;
solutions(i,17)= fSe;
solutions(i,18)= fae;
solutions(i,19)= f0m;
solutions(i,20)= fSm;
solutions(i,21)= fam;
solutions(i,22)= e0m;
solutions(i,23)= eSm;
```

```
solutions(i,24)= eam;

end

xlswrite('NMP_Arbitrageur_Wt.xlsx',solutions);
```

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